Regularity of measures satisfying the annular decay condition

Abstract

Given $\delta \in (0,1]$, we say that a metric measure space (X, d, μ) satisfies the δ-*annular decay condition* (δ-ADC) if

$$\mu\big(B(x,R) \setminus B(x,r)\big) \leq C\Big(\frac{R-r}{R}\Big)^{\delta} \mu\big(B(x,R)\big)$$

for some positive constant $C > 0$, each $x \in X$ and all $0 < r \leq R$.

Under reasonable assumptions on the space, the δ-ADC for some $\delta \in (0,1)$ turns out to be equivalent to the doubling condition. While the existence of singular doubling measures on $\mathbb{R}^n$ is a well known fact with relevant implications in Geometric Function Theory and PDE's, the situation is more delicate if $\delta = 1$. After reviewing some basic background, we will discuss the existence of singular measures on $\mathbb{R}^n$ satisfying the 1-ADC. (Joint work with A. Arroyo).