Title: Strong stability for the Wulff inequality with a crystalline norm

Abstract: Let K be a convex polyhedron and $\mathscr{F}$ its Wulff energy, and let $\mathscr{C}(K)$ denote the set of convex polyhedra close to K whose faces are parallel to those of K. We show that, for sufficiently small ϵ, all ϵ-minimizers belong to $\mathscr{C}(K)$.

As a consequence of this result we obtain the following sharp stability inequality for crystalline norms: There exist $\gamma = \gamma(K, n) > 0$ and $\sigma = \sigma(K, n) > 0$ such that, whenever $|E| = |K|$ and $|E \Delta K| \leq \sigma$, then

$$\mathscr{F}(E) - \mathscr{F}(K^{\mathbf{a}}) \geq \gamma |E \Delta K^{\mathrm{a}}| \qquad \text{for some } K^{\mathbf{a}} \in \mathscr{C}(K).$$

In other words, the Wulff energy $\mathscr{F}$ grows very fast (with power 1) away from the set $\mathscr{C}(K)$. The set $K^{\mathbf{a}} \in \mathscr{C}(K)$ appearing in the formula above can be informally thought as a sort of "projection" of E on the set $\mathscr{C}(K)$.

This is a joint work with Alessio Figalli.