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## Operator algebras that one can see

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The goal of this lecture is threefold: to introduce the fundamentals and the founding idea of noncommutative topology, to explain the basics of graphs and graph algebras, and to exemplify noncommutative topology by the tangible world of graph  $C^*$ -algebras, known as “operator algebras that one can see”. In topology, pushouts are formal recipes for collapsing and gluing topological spaces. For instance, shrinking the boundary circle of a disc to a point yields a sphere, shrinking the equator of a sphere to a point gives two spheres joined at the point, collapsing the boundary of a solid torus to a circle, or gluing two solid tori over their boundaries, produces a three-sphere. In noncommutative topology, such procedures are expressed in terms of pullbacks of  $C^*$ -algebras. Indeed, the GelfandNaimark theorem establishing the anti-equivalence of the category of compact Hausdorff spaces and commutative unital  $C^*$ -algebras turns pushouts into pullbacks. It turns out that one can visualize a pullback of  $C^*$ -algebras of graphs as a pushout of these graphs thus providing much needed intuition to the abstract setting of operator algebras. We will discuss how to make this visualization rigorous by conceptualizing abundant examples from noncommutative topology that lead to a new concept of morphisms of graphs. In particular, we replace the standard idea of mapping vertices to vertices and edges to edges by the more flexible idea of mapping finite paths to finite paths. Time permitting, we will also show how to combine a number of terminating algorithms to prove a theorem about finite paths in finite graphs without loops.

This lecture stems from a semester-long lecture course that I taught at the University of Colorado Boulder. It might be helpful to familiarize oneself with the course home page: [here](#).

It contains lots of studying aids, including complete lecture notes with exercises and movies. A more succinct and purely graph lecture notes can be found [here](#).

**Adnotacja:** wykład będzie prowadzony w języku polskim. Materiały do wykładu (slajdy) będą w języku angielskim.