BURA METHODS FOR SPECTRAL SPACE FRACTIONAL DIFFUSION PROBLEMS

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Fractional diffusion operators appear naturally in various areas in mathematics, physics, ect. The most important property of the related boundary value problems is that they are nonlocal. Let us consider a fractional power of a self-adjoint elliptic operator introduced through its spectral decomposition. It is self-adjoint but nonlocal. Such problems are computationally expensive. Several different techniques were proposed during last decade to localize the nonlocal elliptic operator, thus increasing the space dimension of the original computational domain. An alternative approach is developed in [1, 2, 3, 4]. Let $\mathcal{A}$ be a properly scaled symmetric and positive definite sparse matrix, arising from finite element or finite difference discretization of the initial (standard, local) diffusion problem. Based on best uniform rational approximations (BURA) of $t^\alpha/(1+qt^\alpha)$, $q \geq$ for $t \in [0,1]$, a class of solution methods for solving algebraic systems of linear equations involving $\mathcal{A}^\alpha + q\mathcal{I}$, $0 < \alpha < 1$, is proposed and analysed. Robust error estimates with respect to the condition number $\kappa(\mathcal{A})$ are obtained, showing the exponential convergence rate of BURA methods with respect to the degree of rational approximation k. Although the fractional power of $\mathcal{A}$ is a dense matrix, the algorithm has complexity $O(N)$, where N is the number of unknowns. At this point we assume that some solver of optimal complexity (say multigrid or multilevel) is used for the involved systems with matrices $\mathcal{A} + d_j\mathcal{I}$, $d_j \geq 0$, $d_j \geq 0$, $j = 1, \ldots, k$. This leads at the end to an almost optimal complexity $O(N \log N)$ of the composite algorithm. The presented (up to 3D) numerical tests are focussed on problems with low regularity of the solutions, including cases of adaptive mesh refinement. The comparative analysis demonstrates well expressed advantages of the BURA methods. A unified theoretical explanation of these observations is discussed at the end.

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