Plenary Talks

Monday

Charles Fefferman (Princeton)
Interpolation of Data by Smooth Functions

Let X be your favorite Banach space of functions on $\mathbb{R}^n$. Given a real-valued function f on a (possibly awful) subset E of $\mathbb{R}^n$, we ask:

- How can we decide whether f extends to a function F on $\mathbb{R}^n$, with F belonging to X?
- If such an F exists, then how small can we take its norm in X? Can we make F depend linearly on f? What can we say about the derivatives of F at or near points of E (assuming functions in X have derivatives)?
- If such an F exists, then how small can we take its norm in X? Can we make F depend linearly on f? What can we say about the derivatives of F at or near points of E (assuming functions in X have derivatives)?

Suppose E is finite. Can we compute an F whose norm in X has the smallest possible order of magnitude? How many computer operations does it take? What if we demand merely that F agree approximately with f on E? What if we are allowed to discard a few data points as *outliers*? Which points should we discard?

Tadeusz Iwaniec (Syracuse)
Let a Nonlinear Harmonic Analysis be Revealed through Variational Principles

Sobolev homeomorphisms and their limits are widely studied in Geometric Function Theory (GFT) and mathematical models of Nonlinear Elasticity (NE). It is at the heart of the present lecture to convince you that the weak limits of Sobolev homeomorphisms are legitimate deformations of hyper-elastic materials. As we seek greater knowledge about the energy-minimal deformations in NE, the questions of existence and injectivity (motivated by the principle of non-interpenetration of matter) become ever more quintessential. Nonlinear PDEs and topology of monotone mappings come into play. Theoretical prediction of failure of bodies, caused by cracks, should appeal to both:

Mathematical Analysts and Researchers in the Engineering Fields

In case of the materials with Dirichlet stored-energy, we shall see that cracks propagate along vertical trajectories of the associated Hopf quadratic differential.
I will summarize, in the briefest possible terms, our recent advances with ***Jani Onninen***. It goes back to the concept of Direct Method in the Calculus of Variations introduced by David Hilbert and Stanisław Zaremba – the first President of the Polish Mathematical Society.

Tuesday

Xavier Tolsa (ICREA - Universitat Autònoma de Barcelona)
The regularity problem for the Laplace equation in rough domains

Given a bounded domain $\Omega \subset \mathbb{R}^n$, one says that the L^p-regularity problem is solvable for the Laplace equation in Ω if, given any continuous function f defined in $\partial\Omega$ and the harmonic extension u of f to Ω, the non-tangential operator of the gradient of u can be controlled in L^p norm by the tangential derivative of f in $\partial\Omega$. Up to now this was only known to hold for Lipschitz domains (in some range of p's). In my talk I will explain a recent result with Mihalis Mourgoglou where we show that the L^p-regularity is also solvable (in some range of p's) in a much larger class of domains, such as the so called 2-sided chord-arc domains. For (1-sided) chord arc domains, this result fails, but we provide a positive answer in terms of the gradient of f in the Hajłasz Sobolev space. This solves a problem posed by Carlos Kenig in 1991.

Jan Kristensen (Oxford)

Oscillation and concentration in sequences of PDE constrained maps

It is well-known that for exponents $p > 1$, any L_p-weakly converging sequence of PDE constrained maps admits, up to a subsequence, a decomposition into sequences of PDE constrained maps where one converges in measure (no oscillation) and the other is p-equi-integrable (no concentration). For $p = 1$ the relevant corresponding result concerns weakly* convergent sequences of PDE constrained measures and is false: the oscillation and concentration cannot be separated while simultaneously satisfying the PDE constraint. In this talk we explain how the concentration regardless of the failure of a decomposition result retains its PDE character. The presented results are parts of joint works with Andre Guerra and Bogdan Raita.

Nicola Garofalo (Padova)

Heat kernels for some hybrid evolution equations

The aim of my talk is to construct some explicit heat kernels for a class of hybrid evolution equations which arise in conformal CR geometry and in subelliptic PDEs. By hybrid I mean that the relevant partial differential operator appears in the form $\mathcal{L}_1 + \mathcal{L}_2 - p_t$, but the variables cannot be decoupled. This means that the relative heat kernel cannot be written as the product of the heat kernels of the operators $\mathcal{L}_1 - p_t$ and $\mathcal{L}_2 - p_t$. The approach is completely self-contained, elementary, and it is purely based on PDE methods. The aim is to emphasise the so far unexplored connection of such hybrid equations with the heat kernel of the generalised operator of Ornstein-Uhlenbeck type in the opening of Hörmander's groundbreaking 1967 work on hypoellipticity. This is joint wok with Giulio Tralli.

Wednesday

Jean Van Schaftingen (UCLouvain)

Marcinkiewicz meets Gagliardo and Sobolev: getting weak differentiability from weak-type estimates

I will present characterisations of Sobolev spaces with Marcinkiewicz weak-type estimates for the integrand of the Gagliardo semi-norm, their application to detection of constant functions and repairing the fractional Gagliardo–Nirenberg interpolation at endpoints where it fails and connections with interpolation.

These are joint works with Haim Brezis (Rutgers, Technion Haifa and Sorbonne), Andreas Seeger (University of Wisconsin-Madison), Brian Street (University of Wisconsin-Madison) and Po Lam Yung (Chinese University of Hong Kong and Australian National University).

Alexander Volberg (Michigan State University)

Strange property of positive measures and Carleson embedding on certain graphs with cycles

Carleson embedding theorems often serve as a first building block for interpolation in complex domain, for the theory of Hankel operators, and in PDE. The embedding of certain spaces of holomorphic functions on n-polydisc can be reduced (without loss of information) to the boundedness of weighted multi-parameter dyadic Carleson embedding. We find the necessary and sufficient condition for this Carleson embedding in n-parameter case, when n is 1, 2, or 3. The main tool is the harmonic analysis on graphs with cycles. The answer is quite unexpected and seemingly goes against the well known difference between box and Chang–Fefferman condition that was given by Carleson quilts example of 1974. The main tool is an unexpected combinatorial properties of positive measures on cube in dimensions 1, 2, 3. I will present results obtained jointly by Arcozzi, Holmes, Mozolyako, Psaromiligkos, Zorin-Kranich and myself.

Pekka Koskela (Jyväskylä)
Radial and vertical limits for A_p-weighted Sobolev functions

Let $n \geq 2$. According to results of Uspenskii and Timan, each C^1-function u for which $|\nabla u|^p$ is integrable over $\mathbb{R}^n$ has a unique radial limit along almost every direction precisely when $1 \leq p < n$. A more modern version asserts this for a suitable representative a function in the local Sobolev space $W^{1,1}$, with the same assumption on the integrability of the partial derivatives. Fefferman and Portnov showed that the same conclusion holds for limits along almost all half-lines parallel to the coordinate axes. We describe our recent attempts to generalize these results to the Muckenhoupt A_p-weighted setting. This is joint work with Sylvester Eriksson-Bique and Khanh Nguyen.

Thursday

Alexander Olevskii (Tel Aviv)
Exponential Riesz bases and frames

I will present an introduction to the subject and discuss the following problem: Do exponential Riesz bases and frames exist in the space $L^2(S)$ for every set $S \subset R$ of finite measure? Based on joint works with G.Kozma, S.Nitzan and A.Ulanovskii.

Nicola Gigli (SISSA)
Module-valued measures and applications

Consider a real valued BV function defined on a Riemannian manifold: in which space does its differential live? The answer to this question is typically given by either using coordinates or via polar decomposition. When working on less regular structures, typically neither of these is a priori given, and questions like *what is it the differential of a BV function, or the Hessian of a convex function* are often left unanswered.

In this talk I will present a framework to deal with this sort of problems. A key concept is that of module-valued measures defined over arbitrary metric spaces, a notion that seems to encapsulate several pre-existing ones in metric analysis, including in particular Ambrosio-Kirchheim's metric currents. Applications to BV calculus on RCD spaces will be discussed.

From a joint work with Camillo Brena.

Stefan Wenger (Fribourg)
Uniformization under minimal conditions

We consider the uniformization problem for 2-dimensional geodesic metric surfaces of locally finite Hausdorff measure. We show that there exist parametrizations with good geometric and analytic properties without assuming any further conditions on the space. We furthermore discuss how these parametrizations can be upgraded to quasiconformal or quasisymmetric parametrizations under suitable additional conditions. Our results in particular recover Bonk-Kleiner's quasisymmetric uniformization theorem and Rajala's quasiconformal uniformization theorem for metric surfaces. Based on joint work with Damaris Meier.

Piotr Hajłasz (University of Pittsburgh)

Lusin-type properties of convex functions and convex bodies

My talk is based on a joint paper with Daniel Azagra. We prove that if $f : \mathbb{R}^n \to \mathbb{R}$ is convex and $A \subset \mathbb{R}^n$ has finite measure, then for any $\varepsilon > 0$ there is a convex function $g : \mathbb{R}^n \to \mathbb{R}$ of class $C^{1,1}$ such that $\mathcal{L}^n(\{x \in A : f(x) \neq g(x)\}) < \varepsilon$. As an application we deduce that if $W \subset \mathbb{R}^n$ is a compact convex body then, for every $\varepsilon > 0$, there exists a convex body W_ε of class $C^{1,1}$ such that $\mathcal{H}^{n-1}(\partial W \setminus \partial W_\varepsilon) < \varepsilon$. We also show that if $f : \mathbb{R}^n \to \mathbb{R}$ is a convex function and f is not of class $C^{1,1}_{\text{loc}}$, then for any $\varepsilon > 0$ there is a convex function $g : \mathbb{R}^n \to \mathbb{R}$ of class $C^{1,1}_{\text{loc}}$ such that $\mathcal{L}^n(\{x \in \mathbb{R}^n : f(x) \neq g(x)\}) < \varepsilon$ if and only if f is essentially coercive, meaning that $\lim_{|x|\to\infty} f(x) - \ell(x) = \infty$ for some linear function ℓ. A consequence of this result is that, if S is the boundary of some convex set with nonempty interior (not necessarily bounded) in $\mathbb{R}^n$ and S does not contain any line, then for every $\varepsilon > 0$ there exists a convex hypersurface S_ε of class $C^{1,1}_{\text{loc}}$ such that $\mathcal{H}^{n-1}(S \setminus S_\varepsilon) < \varepsilon$.

Friday

Rafał Latała (UW)

Two-sided bounds on moments of weighted sums of finite Riesz products

Let n_j be a lacunary sequence of integers, such that $n_{j+1}/n_j \geq r$. We investigate linear combinations of the sequence of finite Riesz products $\prod_{j=1}^{k}(1 + \cos(n_j t))$. We show that, whenever the Riesz products are normalized in L_p norm ($p \geq 1$) and when r is large enough, the L_p norm of such a linear combination is equivalent to the ℓ_p norm of the sequence of coefficients. We also discuss generalization of this fact to vector valued L_p spaces. Based on the joint work with Aline Bonami, Piotr Nayar and Tomasz Tkocz.