Local nearrings of order 128 with 2-generated additive groups

I. Raievska, M. Raievska

Institute of Mathematics of NAS of Ukraine, Kyiv, Ukraine

Introduction

There exist 267 non-isomorphic groups of order $64 = 2^6$ from which 53 are 2-generated groups and only 24 of these groups are the additive groups of local nearrings [1]. There exist 2328 non-isomorphic groups of order $128 = 2^7$ from which 162 are 2-generated groups (5 groups are of exponent 64, 18 groups are of exponent 32, 65 groups are of exponent 16, 72 groups are of exponent 8, and 2 groups are of exponent 4).

Nearrings are generalization of associative rings, in which the additive group can be non-abelian, and addition is connected with multiplication by only one distributive law, left or right. In this sense local nearrings are generalization of local rings.

Clearly every associative ring is a nearring and each group is the additive group of a nearring, but not necessarily of a nearring with identity.

Boykett and Nöbauer [2] classified all non-abelian groups of order less than 32 that can be the additive groups of a nearring with identity and found the number of non-isomorphic nearrings with identity on such groups (see also GAP [3] package SONATA [4]).

A study of local nearrings was initiated by Maxson [5] who defined a number of their basic properties and proved in particular that the additive group of a finite zero-symmetric local nearring is a p-group.

A nearring R is called *local*, if the set L of all non-invertible elements of R forms a subgroup of its additive group R^+.

The classification of all nearrings up to certain orders is an open problem. However, the classification of nearrings of higher orders requires much more complex calculations.

The list of all 698 local nearrings of order at most 31 up to isomorphism is provided by the GAP package SONATA; however, classifying nearrings of order 32 is a significant challenge.

The current version of the package LocalNR [6] (not yet redistributed with GAP) contains all 37441 local nearrings of order at most 361, except those of orders 32, 64, 128, 243 and 256. We have already calculated some classes of local nearrings of orders 32 (with 14927685 nearrings), 64 (with 1115947 nearrings) and 243 (with 705105 nearrings).

Let $[n, i]$ be the i-th group of order n in the SmallGroups library in the computer system algebra GAP. We denote by C_n and D_n the cyclic and dihedral groups of order n, respectively.

The library of zero-symmetric local nearrings of order 128 on 2-generated groups can be extracted from [7] using GAP and the package LocalNR.

Theorem 1.

The following 2-generated groups of exponent 4 and only they are the additive groups of zero-symetric local nearrings of order 128:

IdGroup	Structure Description	Number of LNR
[128, 36]	$(C_2 \times ((C_4 \times C_2) \rtimes C_2)) \rtimes C_4$	> 80384
[128, 125]	$(C_4 \times C_4 \times C_2) \rtimes C_4$	> 35040

Theorem 2. [8]

The following 2-generated groups of exponent 16 and only they are the additive groups of zero-symmetric local nearrings of order 128:

IdGroup	Structure Description	Number of LNR
[128, 42]	$C_{16} \times C_8$	> 134754
[128, 43]	$C_{16} \rtimes C_8$	> 133866
[128, 44]	$C_8 \rtimes C_{16}$	> 145648
[128, 46]	$((C_{16} \times C_2) \rtimes C_2) \rtimes C_2$	> 24704
[128, 47]	$((C_{16} \times C_2) \rtimes C_2) \rtimes C_2$	252928
[128, 52]	$((C_{16} \rtimes C_2) \rtimes C_2) \rtimes C_2$	> 115840
[128, 53]	$((C_{16} \rtimes C_2) \rtimes C_2) \rtimes C_2$	> 277248
[128, 54]	$(C_4 \times C_2) \rtimes C_{16}$	> 82944
[128, 55]	$(C_4 \times C_2).((C_4 \times C_2) \rtimes C_2) = (C_4 \times C_2).(C_8 \times C_2)$	640
[128, 59]	$C_4.((C_2 \times C_2 \times C_2) \rtimes C_4) = (C_4 \times C_2).(C_8 \times C_2)$	> 13056
[128, 99]	$C_8 \rtimes C_{16}$	> 29248
[128, 102]	$C_8 \rtimes C_{16}$	> 5376
[128, 106]	$(C_{16} \times C_2) \rtimes C_4$	> 2808
[128, 107]	$(C_{16} \times C_2) \rtimes C_4$	> 16460
[128, 108]	$(C_{16} \rtimes C_2) \rtimes C_4$	> 1344
[128, 109]	$(C_{16} \rtimes C_2) \rtimes C_4$	> 2344

Theorem 3.

There exist 389976 zero-symetric local nearrings on 2-generated additive groups of exponent 32 of order 128:

IdGroup	Structure Description	Number of LNR
[128, 128]	$C_{32} \times C_4$	48968
[128, 129]	$C_{32} \rtimes C_4$	48968
[128, 131]	$(C_{32} \times C_2) \rtimes C_2$	144016
[128, 132]	$(C_{32} \rtimes C_2) \rtimes C_2$	23936
[128, 153]	$C_4 \rtimes C_{32}$	118968
[128, 154]	$C_{16}.D_8 = C_4.(C_{16} \times C_2)$	5120

Theorem 4.

There exist 1024 zero-symetric local nearrings on 2-generated additive groups of exponent 64 of order 128:

IdGroup	Structure Description	Number of LNR
[128, 159]	$C_{64} \times C_2$	512
[128, 160]	$C_{64} \rtimes C_2$	512

References

[1] Raievska I., Raievska M., Sysak Ya. Local nearrings of order 64. International mathematical conference "Groups and Actions: Geometry and Dynamics, dedicated to the memory of professor Vitaly Sushchanskyy" (2016, December 19–22, Kyiv, Ukraine). Book of abstracts. — Kyiv, 2016. — P. 43.

[2] Boykett T. H. H., Nöbauer C. A class of groups which cannot be the additive groups of near-rings with identity. Contributions to general algebra, 10 (Klagenfurt, 1997). — Klagenfurt: Heyn, 1998. — P. 89—99.

[3] The GAP Group, GAP – Groups, Algorithms, and Programming, Version 4.11.0; 2020 https://www.gap-system.org

[4] Aichinger E., Binder F., Ecker Ju., Mayr P. and Nöbauer C. *SONATA — System of nearrings and their applications, GAP package, Version 2.8*; 2015. http://www.algebra.uni-linz.ac.at/Sonata/

[5] Maxson C. J. On local near-rings. Math. Z. — 1968. — 106. — P. 197–205.

[6] Raievska, I., Raievska, M. and Sysak, Y., LocalNR, Package of local nearrings, Version 1.0.3 (2021) (GAP package), https://gap-packages.github.io/LocalNR/

[7] Iryna Raievska, Maryna Raievska, & Yaroslav Sysak. (2022). DatabaseEndom128: (v0.1) [Data set]. Zenodo, https://doi.org/10.5281/zenodo.6538441

[8] Raievska I., Raievska M. Local nearrings with additive groups of order 128 and exponent 16. Conference of young scientists "Pidstryhach readings — 2022" (2022, May 25–27, Lviv, Ukraine) http://www.iapmm.lviv.ua/chyt2022/abstracts/

Acknowledgements

This work was partially supported by the Polish Academy of Sciences (PAN) and National Academy of Sciences (NAS). The authors would like to thank the Institute of Mathematics of the Polish Academy of Sciences for hosting after evacuation from Ukraine.

Contact Information

- Web: http://www.imath.kiev.ua/ algebra/Raievska_I
- Email: raeirina@imath.kiev.ua
- Web: http://www.imath.kiev.ua/ algebra/Raievska_M
- Email: raemarina@imath.kiev.ua