

# Complexity of polynomial progressions and the polynomial Szemerédi theorem

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The polynomial Szemerédi theorem of Bergelson and Leibman is a central result at the interface between ergodic theory and additive combinatorics, extending earlier results of Szemerédi and Furstenberg on arithmetic progressions. It states that each dense subset of integers contains certain polynomial configurations. Using a correspondence principle introduced by Furstenberg, the theorem can be deduced from an ergodic-theoretic result on the convergence of multiple ergodic averages with polynomial iterates. The limiting behaviour of such averages has been an object of intense study by ergodic theorists and additive combinatorists alike. In this talk, we describe the nature of the limit of averages related to certain families of polynomial configurations, for which little has been known previously. In doing so, we define, discuss and connect various notions of complexity of polynomial configurations. As a consequence, we derive certain multiple recurrence results that extend the classical recurrence theorem of Khintchine.