

Transitions from one- to two-dimensional dynamics

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Studies have shown that the dimension of a dynamical system restricts the possible dynamical behavior of a system. Here, we investigate how the restrictions relief as the dimension increases. In particular, we introduce a topological model which applies to the Hénon [H] and the Lozi [L] families and view the families as perturbations of the unimodal and the tent families in two dimensions respectively. We use the model to explain the following:

1. A two dimensional system can have infinitely many sinks [N1, N2, R], where as a one dimensional system can not [S].
2. The kneading theory [MT] breaks down in the Hénon and the Lozi families.
3. There are no Fibonacci maps [LM] in two dimensions.

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