OBSTRUCTIONS TO REALIZING TOROIDAL SETS AS ATTRACTORS

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It is well known that (asyptotically stable) attractors can have a very complicated topological structure. This prompts the following question: given a compact set $K \subseteq \mathbb{R}^3$, what are the topological obstructions to the existence of a homeomorphism h of $\mathbb{R}^3$ having K as a (local) attractor?

A compact set $K \subseteq \mathbb{R}^3$ is cellular when it has a neighbourhood basis of cells. Cellular sets can always be realized as attractors. The next natural step is to consider sets K which have a neighbourhood basis comprised of solid tori. These we call *toroidal*, and some examples are generalized knotted solenoids and some wild knots. Unlike cells, solid tori can be knotted and also wind inside each other. Guided by these two ideas we associate to each toroidal set K two magnitudes $g(K)$ and $\mathcal{N}(K)$ which capture the amount of knottedness and "self–winding" of the set. If a toroidal set can be realized as an attractor then both magnitudes have a certain finiteness property. We will construct examples of toroidal sets which do not have that property and therefore cannot be realized as attractors.

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