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Transitivity and mixing properties for expanding Lorenz maps on the interval

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Abstract. Suppose that $f : [0,1] \to [0,2]$ is a continuous strictly increasing function which is differentiable on $(0,1) \setminus F$ where F is a finite set. Moreover, assume that $\beta := \inf f' := \inf_{x \in (0,1) \setminus F} f'(x) > 1$. This implies that there is a unique $c \in (0,1)$ with $f(c) = 1$. Define $T_f x := f(x) - \lfloor f(x) \rfloor$, where $\lfloor y \rfloor$ is the largest integer smaller or equal to y. Maps of this kind are called expanding Lorenz maps. Note that T_f is a piecewise monotone map, but it has a discontinuity at c.

Topological transitivity and topological mixing of T_f are investigated in the case $\beta \geq \sqrt[3]{2}$. If $\beta \geq \sqrt[3]{2}$ and $f(0) \geq \frac{1}{\beta+1}$ the map T_f is topologically transitive. Furthermore it is also topologically mixing except in the case $f(x) = \sqrt[3]{2}x + \frac{2+\sqrt[3]{4}-2\sqrt[3]{2}}{2}$ for all $x \in [0,1]$.

Better results are obtained in the special case $f(x) = \beta x + \alpha$. Here one can completely describe the set of all (β, α) with $\sqrt[3]{2} \leq \beta \leq 2$ and $0 \leq \alpha \leq 2 - \beta$ such that T_f is topologically transitive. All of them except three special cases are also topologically mixing.

Glendinning called a map T_f locally eventually onto if for every nonempty open $U \subseteq [0,1]$ there are open intervals $U_1, U_2 \subseteq U$ and there are $n_1, n_2 \in \mathbb{N}$ such that ${T_f}^{n_1}$ maps U_1 homeomorphically to $(0,c)$ and ${T_f}^{n_2}$ maps U_2 homeomorphically to $(c,1)$. The map T_f renormalizable if there are $0 \leq u_1 < c < u_2 \leq 1$ and $l, r \in \mathbb{N}$ with $l + r \geq 3$ such that ${T_f}^l$ is continuous on (u_1, c), ${T_f}^r$ is continuous on (c, u_2), $\lim_{x \to c^-} {T_f}^l x = u_2$ and $\lim_{x \to c^+} {T_f}^r x = u_1$. One can find an example of a renormalizable and locally eventually onto expanding Lorenz map. Nonetheless a condition closely related to "locally eventually onto" is given, and it is shown that this condition is equivalent to T_f is not renormalizable.

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