First Dynamical Systems Summer Meeting

Będlewo, 16-20 August 2021

Abstracts of talks

(online talks are marked in blue)

Rigidity of topological entropy of boundary maps associated to Fuchsian groups
Adam Abrams

Given a closed, orientable surface of constant negative curvature, we study a family of generalized Bowen–Series boundary maps, with each map defined for a particular fundamental polygon for the surface and a particular multi-parameter. We prove the following rigidity result: the topological entropy is constant and depends only on the genus of the surface. This is in contrast to a previous result that measure-theoretic entropy varies greatly within Teichmüller space. We give explicit formulas for both of these entropies. The proofs of rigidity use conjugation to maps of constant slope. This work is joint with Svetlana Katok and Ilie Ugarcovici.

Metric Versus Topological Receptive Entropy of Semigroup Actions
Andrzej Biś

We study the receptive metric entropy for semigroup actions on probability spaces, inspired by a similar notion of topological entropy introduced by Hofmann and Stoyanov (Adv. Math. 115:54–98, 1995). We analyze its basic properties and its relation with the classical metric entropy. In the case of semigroup actions on compact metric spaces we compare the receptive metric entropy with the receptive topological entropy looking for a Variational Principle. With this aim we propose several characterizations of the receptive topological entropy. Finally we introduce a receptive local metric entropy inspired by a notion by Bowen generalized in the classical setting of amenable group actions by Zheng and Chen, and we prove partial versions of the Brin–Katok Formula and the local Variational Principle. The talk is based on a joint paper [1] with Dikran Dikranjan, Anna Giordano Bruno, and Luchezar Stoyanov.

References

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Beyond topological hyperbolicity
Welington Cordeiro

We discuss the dynamics beyond topological hyperbolicity considering homeomorphisms satisfying the shadowing property and generalizations of expansivity. First of all, we will talk about some of these generalizations of expansivity and show some interesting examples. In particular, we will define *entropy expansivity*, *N-expansivity* and *measure expansivity* and we will give an overview of the results and we will have a discussion about a Spectrum Decomposition Theorem for these systems.

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Towards the understanding of inhomogeneities in strange attractors
Jernej Činč

Brown-Barge-Martin embeddings of inverse limits provide a natural way to construct curious examples of strange attractors arising from homeomorphisms on manifolds of dimension at least two. In the recent years we were building towards the better understanding of inhomogeneities of such strange attractors, starting with the ones arising from unimodal interval maps [1]. In this talk I will review the part of the work that was done on inverse limits of one-dimensional manifolds [2]. Namely, I will show that in such a setting we have complete understanding of basic types of inhomogeneities (i.e. folding points and endpoints) through the dynamics of bonding maps when these are piecewise monotone and locally eventually onto. If time permits I will also discuss work in progress [3] that generalises and applies these results. Joint work with Ana Anušić (University of São Paulo).

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[2] A. Anušić, J. Činč, *Inhomogeneities in chainable continua*, Fundamenta Mathematicae **254** (2021) 69–98. (Appendix written by Henk Bruin)
[3] A. Anušić, J. Činč, *Solenoidal and non-solenoidal points in one-dimesnional attractors*, in preparation.

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Haagerup property and Kazhdan pairs in terms of ergodic infinite measure preserving actions online
Alexandre Danilenko

It is shown that a locally compact second countable group G has the Haagerup property if and only if there exists a sharply weak mixing 0-type measure preserving free G-action $T = (T_g)_{g\in G}$

on an infinite σ-finite standard measure space (X, μ) admitting a T-Følner sequence (i.e. a sequence $(A_n)_{n=1}^{\infty}$ of measured subsets of finite measure such that $A_1 \subset A_2 \subset \cdots, \bigcup_{n=1}^{\infty} A_n = X$ and

$$\lim_{n\to\infty} \sup_{g\in K} \frac{\mu(T_g A_n \triangle A_n)}{\mu(A_n)} = 0$$

for each compact $K \subset G$). A pair of groups $H \subset G$ has property (T) if and only if there is a μ-preserving G-action S on X admitting an S-Følner sequence and such that $S \upharpoonright H$ is weakly mixing. These refine some recent results by Delabie-Jolissaint- Zumbrunnen and Jolissaint.

Levels of distributional chaos for Turing Machines
Mauricio Díaz

A Turing machine can be represented as a dynamic system, consisting of two cells, in which there is an infinite tape and a head that act as pointers to describe a certain sequence around given instructions. In this work, we are going to introduce some levels of distributional chaos with respect some increasing sequences through Furstenberg's Block family. Latter, we'll introduce one example, where the system is DC2 but not DC1.

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Recurrence for smooth curves in the moduli space of translation surfaces
Krzysztof Frączek

My talk is a kind of review of problems and recent results regarding smooth curves in the moduli space of translation surfaces and Teichmüller positive semi-orbits starting from such curves. I plan to present some abstract results about the recurrence or equidistribution of Teichmüller positive semi-orbits starting from almost every element of the curve. The main part of the talk are applications that motivate abstract results. Two main applications relate to billiards on nibbled ellipses and impact Hamiltonian systems.

Convergence of renormalizations and rigidity of multicritical circle maps
Igors Gorbovickis

A multicritical circle map is a smooth homeomorphism of the circle with several critical points of odd integer order. We prove exponential convergence of renormalizations for C^3-smooth multicritical circle maps with the same combinatorics and with rotation numbers of bounded type. As a corollary, we obtain a $C^{1+\alpha}$-rigidity theorem for such maps. This is joint work with Michael Yampolsky.

Group actions with discrete spectrum and their amorphic complexity
Maik Gröger

Amorphic complexity, originally introduced for integer actions, is a topological invariant which measures the complexity of dynamical systems in the regime of zero entropy. We will introduce its definition for actions by locally compact σ-compact amenable groups on compact metric spaces. Further, we will illustrate some of its basic properties and show why it is tailor-made to study strictly ergodic group actions with discrete spectrum and continuous eigenfunctions. This class of actions includes, in particular, Delone dynamical systems related to regular model sets obtained via cut and project schemes (CPS). Finally, for these kind of Delone dynamical systems we present sharp upper bounds on amorphic complexity utilizing basic properties of the corresponding CPS.

This is joint work with G. Fuhrmann, T. Jäger and D. Kwietniak.

Entropy beyond actions of uniform lattices
Till Hauser

Measure theoretical entropy is an intensely studied concept with various applications and interpretations. For actions of non-discrete groups, such as $\mathbb{R}^d$ it can be defined by computing the entropy with respect to a uniform lattice, such as $\mathbb{Z}^d$. Nevertheless, there exist (metrizable and σ-compact) locally compact Abelian groups, such as the additive group of p-adic numbers, that do not contain uniform lattices. In this talk we explore two non-equivalent notions of entropy, which both generalize the notion of entropy from the setting of discrete amenable groups to the setting of unimodular amenable groups. The first concept is defined by using the concept of (thin) Følner nets from [1]. The second concept will be defined by replacing the uniform lattice by a weaker structure, called a Delone set, which exists in every unimodular amenable group. This concept generalizes the concept considered in [2]. We relate these notions to the respective notions of topological pressure, present a link to naive entropy and proof respective versions of Goodwyn's half of the variational principle. In collaboration with Friedrich Martin Schneider.

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Tail invariant measures on Bratteli diagrams and their generalizations
Olena Karpel

During the last decades, Bratteli diagrams turned out to be a very powerful tool for the study of dynamical systems not only on a measure space but also on Cantor and Borel spaces. This is due to the fact that homeomorphisms of a Cantor space and Borel automorphisms of a standard Borel space can be represented as Vershik maps acting on the path spaces of corresponding Bratteli diagrams. Various properties of the transformations become more transparent when one deals with corresponding Bratteli-Vershik dynamical systems. In particular, this observation concerns invariant measures and their supports, minimal components of the transformation, structure of its orbits, etc. We will discuss some natural methods for the study of the set of invariant measures in Cantor and Borel dynamics based on the structure of the underlying diagram. These methods also work even for Bratteli diagrams that do not support any Vershik map. We consider the so-called generalized Bratteli diagrams in Borel dynamics, give sufficient conditions of unique ergodicity for such diagrams and consider the supports of tail invariant measures. The talk is based on a joint work with Sergey Bezuglyi, Palle E.T. Jorgensen and Shrey Sanadhya.

Complexity of polynomial progressions and the polynomial Szemerédi theorem
Borys Kuca

The polynomial Szemerédi theorem of Bergelson and Leibman is a central result at the interface between ergodic theory and additive combinatorics, extending earlier results of Szemerédi and Furstenberg on arithmetic progressions. It states that each dense subset of integers contains certain polynomial configurations. Using a correspondence principle introduced by Furstenberg, the theorem can be deduced from an ergodic-theoretic result on the convergence of multiple ergodic averages with polynomial iterates. The limiting behaviour of such averages has been an object of intense study by ergodic theorists and additive combinatorists alike. In this talk, we describe the nature of the limit of averages related to certain families of polynomial configurations, for which little has been known previously. In doing so, we define, discuss and connect various notions of complexity of polynomial configurations. As a consequence, we derive certain multiple recurrence results that extend the classical recurrence theorem of Khintchine.

Entropy rate of product of independent processes
Joanna Kułaga-Przymus

We study the multiplicative version of the classical Furstenberg's filtering problem, where instead of the sum $\mathbf{X}+\mathbf{Y}$ one considers the product $\mathbf{X}\cdot\mathbf{Y}$ ($\mathbf{X}$ and $\mathbf{Y}$ are bilateral, real, finitely-valued, stationary independent processes, $\mathbf{Y}$ is taking values in $\{0,1\}$). We provide formulas for $\mathbf{H}\left(\mathbf{X}\cdot\mathbf{Y}\mid\mathbf{Y}\right)$. As a consequence, we show that if $\mathbf{H}\left(\mathbf{X}\right)>\mathbf{H}\left(\mathbf{Y}\right)=0$ and $\mathbf{X}\amalg\mathbf{Y}$, then $\mathbf{H}\left(\mathbf{X}\cdot\mathbf{Y}\right)<\mathbf{H}\left(\mathbf{X}\right)$ (and thus $\mathbf{X}$ cannot be filtered out from $\mathbf{X}\cdot\mathbf{Y}$) whenever $\mathbf{X}$ is not bilaterally deterministic, $\mathbf{Y}$ is ergodic and $\mathbf{Y}$ first return to 1 can take arbitrarily long with positive probability. On the other hand, if almost surely $\mathbf{Y}$ visits 1 along an infinite arithmetic progression of a fixed difference (with possibly some more visits in between) then we can find $\mathbf{X}$ that is not bilaterally deterministic and such that $\mathbf{H}\left(\mathbf{X}\cdot\mathbf{Y}\right)=\mathbf{H}\left(\mathbf{X}\right)$. We apply these results to $\mathscr{B}$-free systems and partly settle some open problems related to their sets of invariant measures.

Based on joint work with Michał D. Lemańczyk.

On Furstenberg systems of some aperiodic multiplicative functions
Mariusz Lemańczyk

Studying arithmetic properties of multiplicative functions through the so-called Furstenberg systems became a powerful and fruitful ergodic tool when dealing with the Sarnak and Chowla conjectures. The Chowla conjecture from 1965, originally formulated for the Liouville function, was expected to hold in the class of aperiodic multiplicative functions in the sense that such functions have precisely one Furstenberg system, and this system is "as random as possible", cf. Elliot's conjecture from 1990. In 2015 Matomäki, Radziwiłł and Tao gave a counterexample to Elliot's conjecture by constructing aperiodoc multiplicative functions (bounded by 1) for which (already) the Chowla conjecture of order 2 fails. During the talk I will try to describe recent results concerning a variety of Furstenberg systems for Matomäki, Radziwiłł, Tao's functions, in particular, showing that the Chowla conjecture holds for them along some subsequences and disproving a recent conjecture by Frantzikinakis and Host. The talk is based on my joint work with Alex Gomilko and Thierry de la Rue.

On hyperbolic sets of polynomials
Genadi Levin

We prove that the intersection of orbits of small Julia sets for an infinitely-renormalizable quadratic polynomial contains no hyperbolic sets. Joint work with Feliks Przytycki.

Rotated odometers
Olga Lukina

We consider infinite interval exchange transformations (IETs) obtained as compositions of finite IETs and the von Neumann-Kakutani map, called rotated odometers. Such systems arise as first return maps of rational flows on translation surfaces of infinite genus with finite number of ends. Although very simple to define, rotated odometers exhibit surprisingly diverse behavior. We study the dynamical and ergodic properties of rotated odometers by means of an associated Bratteli-Vershik system and prove a few classification results about the factors of the unique aperiodic minimal subsystem of a rotated odometer. This is joint work with Henk Bruin.

Travel in an infinite desert online
Michał Misiurewicz

When we tried to compute the coarse entropy for a class of systems given by homeomorphisms of the nonnegative reals, we encountered a problem that can be stated in terms of a travel in an infinite desert.

A caravan traverses an infinite desert studded with oases. It can rest indefinitely at each oasis. Given the sequence of the oases' locations, how does the number of the caravan's itineraries grow with time? We show that the growth is exponential when the oasis sequence is asymptotically linear, and subexponential when the oasis sequence is superlinear. Moreover, the growth has to be superpolynomial, but can be barely so.

This is joint work with William Geller.

Ergodic optimization and multifractal formalism of Lyapunov exponents
Reza Mohammadpour

In this talk we discuss ergodic optimization and multifractal behavior of Lyapunov exponents for matrix cocycles. We show that the restricted variational principle holds for generic cocycles over mixing subshifts of finite type and that the Lyapunov spectrum is equal to the closure of the set where the entropy spectrum is positive for such cocycles. Moreover, we show the continuity of the lower joint spectral radius for linear cocycles under the assumption that linear cocycles satisfy a cone condition.

We consider a subadditive potential Φ. We obtain that for $t \to \infty$ any accumulation point of a family of equilibrium states of $t\Phi$ is a maximizing measure and that the Lyapunov exponent and entropy of equilibrium states for $t\Phi$ converge in the limit $t \to \infty$ to the maximal Lyapunov exponent and entropy of maximizing measures. Moreover, we show that if a 2×2 one-step cocycle satisfies pinching and twisting conditions and there exist strictly invariant cones whose images do not overlap on the Mather set then the Lyapunov-maximizing measures have zero entropy.

Transitions from one- to two-dimensional dynamics
Dyi-Shing Ou

Studies have shown that the dimension of a dynamical system restricts the possible dynamical behavior of a system. Here, we investigate how the restrictions relief as the dimension increases. In particular, we introduce a topological model which applies to the Hénon [H] and the Lozi

[L] families and view the families as perturbations of the unimodal and the tent families in two dimensions respectively. We use the model to explain the following:

(1) A two dimensional system can have infinitely many sinks [N1, N2, R], where as a one dimensional system can not [S].
(2) The kneading theory [MT] breaks down in the Hénon and the Lozi families.
(3) There are no Fibonacci maps [LM] in two dimensions.

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On stability of two dimensional non trivial dynamics online
Liviana Palmisano

I will discuss old and new developments in the aim of understanding stability of non trivial attractors in two dimensional dynamical systems. An example of non trivial dynamics are maps with coexisting infinitely many sinks. It has been shown that the infinitely many sinks survive in codimension two manifolds. The same happens for period doubling attractors. I will introduce other quite general type of non trivial dynamics which also survive along finite codimensional manifolds. This gives rise to laminations of topological classes.

Exponential growth of translation surfaces
Mark Pollicott

We will consider asymptotic properties of translation surfaces (or flat surfaces) X. These surfaces X consist of compact surfaces with a flat metric except at, possibly, a finite number of points (with cone singularities). We will describe a notion of "entropy" described in terms of the rate of growth of circles. In particular, we will be interested in the asymptotic growth of the length of this curve and its distribution on X.

This is joint work with P. Colognese.

Transitivity and mixing properties for expanding Lorenz maps on the interval
Peter Raith

Suppose that $f : [0, 1] \to [0, 2]$ is a continuous strictly increasing function which is differentiable on $(0, 1) \setminus F$ where F is a finite set. Moreover, assume that $\beta := \inf f' := \inf_{x\in(0,1)\setminus F} f'(x) > 1$. This implies that there is a unique $c \in (0, 1)$ with $f(c) = 1$. Define $T_f x := f(x) - \lfloor f(x)\rfloor$, where $\lfloor y\rfloor$ is the largest integer smaller or equal to y. Maps of this kind are called expanding Lorenz maps. Note that T_f is a piecewise monotone map, but it has a discontinuity at c.

Topological transitivity and topological mixing of T_f are investigated in the case $\beta \geq \sqrt[3]{2}$. If $\beta \geq \sqrt[3]{2}$ and $f(0) \geq \frac{1}{\beta+1}$ the map T_f is topologically transitive. Furthermore it is also topologically mixing except in the case $f(x) = \sqrt[3]{2}x + \frac{2+\sqrt[3]{4}-2\sqrt[3]{2}}{2}$ for all $x \in [0, 1]$.

Better results are obtained in the special case $f(x) = \beta x + \alpha$. Here one can completely describe the set of all (β, α) with $\sqrt[3]{2} \leq \beta \leq 2$ and $0 \leq \alpha \leq 2 - \beta$ such that T_f is topologically transitive. All of them except three special cases are also topologically mixing.

Glendinning called a map T_f locally eventually onto if for every nonempty open $U \subseteq [0,1]$ there are open intervals $U_1, U_2 \subseteq U$ and there are $n_1, n_2 \in \mathbb{N}$ such that ${T_f}^{n_1}$ maps U_1 homeomorphically to $(0,c)$ and ${T_f}^{n_2}$ maps U_2 homeomorphically to $(c,1)$. The map T_f renormalizable if there are $0 \leq u_1 < c < u_2 \leq 1$ and $l, r \in \mathbb{N}$ with $l + r \geq 3$ such that ${T_f}^l$ is continuous on (u_1, c), ${T_f}^r$ is continuous on (c, u_2), $\lim_{x \to c^-} {T_f}^l x = u_2$ and $\lim_{x \to c^+} {T_f}^r x = u_1$. One can find an example of a renormalizable and locally eventually onto expanding Lorenz map. Nonetheless a condition closely related to "locally eventually onto" is given, and it is shown that this condition is equivalent to T_f is not renormalizable.

Obstructions to realizing toroidal sets as attractors online

Jaime Jorge Sánchez-Gabites

It is well known that (asyptotically stable) attractors can have a very complicated topo- logical structure. This prompts the following question: given a compact set $K \subset \mathbb{R}^3$, what are the topological obstructions to the existence of a homeomorphism h of $\mathbb{R}^3$ having K as a (local) attractor? A compact set $K \subset \mathbb{R}^3$ is cellular when it has a neighbourhood basis of cells. Cellular sets can always be realized as attractors. The next natural step is to consider sets K which have a neighbourhood basis comprised of solid tori. These we call toroidal, and some examples are generalized knotted solenoids and some wild knots. Unlike cells, solid tori can be knotted and also wind inside each other. Guided by these two ideas we associate to each toroidal set K two magnitudes $g(K)$ and $\mathcal{N}(K)$ which capture the amount of knottedness and "self– winding" of the set. If a toroidal set can be realized as an attractor then both magnitudes have a certain finiteness property. We will construct examples of toroidal sets which do not have that property and therefore cannot be realized as attractors. Joint work with H. Barge.

Dimension estimates for C^1 iterated function systems online

Károly Simon

We introduce a generalized transversality condition (GTC) for parameterized families of C^1 IFSs, on $\mathbb{R}^d$ and show that if the GTC is satisfied then the dimensions of the attractor and of the ergodic invariant measures are given by the singularity dimension, and Lyapunov dimension respectively. Then we verify the GTC for some parametrized families of C^1 IFSs on $\mathbb{R}^d$.

The talk is based on a recent paper joint with De-Jun Feng.

Densely branching trees as models for Hénon-like and Lozi-like attractors

Sonja Štimac

We show that Hénon-like and Lozi-like maps on their strange attractors are conjugate to shift homeomorphisms on inverse limits (also called natural extensions) of maps on metric trees with dense set of branch points. For Hénon maps this applies to Benedicks-Carleson positive Lebesgue measure parameter set, and sheds more light onto the result of Barge from 1987, who showed that there exist parameter values for which Hénon maps on their attractors are not natural extensions of any maps on branched 1-manifolds. For Lozi maps the result applies to an open set of parameters given by Misiurewicz in 1980. In general, under mild dissipation, introduced by Crovisier and Pujals in 2017, conjugacy holds if the attractor does not have an arc in common with a stable manifold, and the density of branch points holds when the attractor is a homoclinic class. Our result can be seen as a generalization of a classical result of Williams from 1967 to the non-uniformly hyperbolic world. To the best of our knowledge, these are the first examples of canonical two-parameter families of attractors in the plane for which such a 1-dimensional locally connected model is given, tying together topology and dynamics of these attractors. Joint work with Jan P. Boroński, AGH University of Science and Technology, boronski@agh.edu.pl.

The fundamental inequality for random walks on cocompact Fuchsian groups
Giulio Tiozzo

A recurring question in the theory of random walks on hyperbolic spaces asks whether the hitting (harmonic) measures can coincide with measures of geometric origin, such as the Lebesgue measure. This is also related to the inequality between entropy and drift, going back to Guivarc'h and called the *fundamental inequality* by Vershik.

For finitely-supported random walks on cocompact Fuchsian groups with sym- metric fundamental domain, we prove that the hitting measure is singular with respect to Lebesgue measure; moreover, its Hausdorff dimension is strictly less than 1.

Along the way, we prove a purely geometric inequality for geodesic lengths, strongly reminiscent of the Anderson-Canary-Culler-Shalen inequality for free Kleinian groups.

Joint with P. Kosenko.

The shrinking target problem and recurrence for generic self-affine sets
Sascha Troscheit

The shrinking target problem in dynamical systems studies the sizes of points that fall into a specified set of "targets" infinitely often. These limsup sets have attracted a lot of attention over the past few years, but not much is known for non-conformal systems. Koivusalo and Ramirez proved that the Hausdorff dimension of recurrent sets for a shrinking (cylinder) target with fixed centre is generically given by the zero of a modified pressure. In this talk we present a new result which removes many of their technical assumptions and provides a more general formula for the Hausdorff dimension of recurrent sets for shrinking (cylinder) target sets with arbitrary centres. Joint work with Balázs Bárány.