

ON SOME IDEALS DEFINED BY DENSITY TOPOLOGIES ON THE REAL LINE AND IN THE CANTOR SPACE.

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The condition (in fact, an ideal) (a) was first introduced in [MG] and investigated in many papers ([GS], [FN1], [FN2], [N]). We can define it using the density topology τ_d , namely $(a) = \{A \subseteq \mathbb{R} : \forall U \in \tau_d \setminus \{\emptyset\} \exists W \in \tau_e U \cap W \neq \emptyset \wedge U \cap W \cap A = \emptyset\}$ (where τ_e is the standard topology on \mathbb{R}).

- It is known that $(a) \subsetneq \text{ND} \cap \mathcal{N}$, where ND is the collection of nowhere dense sets and \mathcal{N} is the sigma ideal of Lebesgue null sets. We prove that in the case of category density topology (see [PWBW]) instead the density topology we obtain the ideal ND.
- It is known that (a) is equal to intersection of the collections of nowhere dense sets for all topologies between τ_e and τ_d . We prove that (a) cannot be represented as a finite intersection of the collections of nowhere dense sets for some topologies between τ_e and τ_d .
- We consider generalized version of the collection (a) for another topologies and collections and we give a construction of a G_δ set which is a counterexample to the inclusion $(a)(\tau_e, \mathcal{B}_{\text{EL}}) \subseteq (a)(\tau_e, \tau_{\text{EL}})$ (where τ_{EL} and \mathcal{B}_{EL} denote the Ellentuck topology and its standard base, respectively).

We also discuss an application of the notion of Marczewski - Burstin representability here, and we consider the another collection (a') and the problem posed in the article [FG] whether the ideal generated by (a') is equal to (a) . Several another open problems will be posed in this context.

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