ON A LACZKOVICH PROPERTY

ELIZA JABŁOŃSKA
(JOINT WORK WITH TARAS BANAKH)

Laczkovich [5] proved that every nonopen analytic subgroup of the real line can be covered by countably many closed sets of Lebesgue measure zero. We generalize the result in the following way:

Any nonopen analytic subgroup of a locally compact topological group X can be covered by countably many closed sets of Haar measure zero in X.

We also show that this theorem does not extend to non-locally compact groups; i.e. the Polish Abelian group $\mathbb{Z}^\omega$ contains a Borel subgroup which is Haar-null and Haar-meager but cannot be covered by countably many closed Haar-meager sets, see [2, Example 8.1]. The notions of Haar-null sets and Haar-meager sets were introduced by Christensen [3] and Darji [4], respectively.

Finally, we show that an example distinguishing the σ-ideal generated by closed Haar-meager sets can be found among Borel linear subspaces of the countable product of lines.

References

[1] I. Banakh, T. Banakh, E. Jabłońska, *Products of K-analytic sets in locally compact groups and Kuczma-Ger classes*, Axioms 11 (2022), 65.
[2] T. Banakh, S. Głąb, E. Jabłońska, J. Swaczyna, *Haar-$\mathcal{I}$ sets: looking at small sets in Polish groups through compact glasses*, Dissert. Math. 564 (2021), 105 pp.
[3] J.P.R. Christensen, *On sets of Haar measure zero in abelian Polish groups*, Israel J. Math. 13 (1972), 255–260.
[4] U.B. Darji, *On Haar meager sets*, Topology Appl. 160 (2013), 2396–2400.
[5] M. Laczkovich, *Analytic subgroups of the reals*, Proc. Amer. Math. Soc. 126 (6) (1998), 1783–1790.

AGH University of Science and Technology, Faculty of Applied Mathematics, Mickiewicza 30, 30-059 Kraków, Poland
Email address: elizajab@agh.edu.pl