A few remarks on Cantorvals

Mateusz Kula

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Given a compact set $K \subseteq \mathbb{R}$ one might consider the family of all non-trivial components of K together with a family of all components of $\mathbb{R} \setminus K$. The sum of these families is naturally equipped with a linear order. We show that the nature of this order uniquely determines the set K up to a homeomorphism, which is actually an automorphism of $\mathbb{R}$. We apply this result to prove a new characterization of the so-called Cantorval. Topological characterizations of the Cantorval may be found in [2, Theorem 1], [4, Theorem 1], [3, Theorem 14] and [1, Theorem 21.20].

References

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