Real Analytic Inspiration
-from Continuous Images of Bernstein Sets
to (non-)Productable Continua

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A. Osipov asked if for every Bernstein set $B \subseteq \mathbb{R}$ the real line can be covered by a countable family of sets each of which is a continuous image of B.

A topological space X is *fiberable* if there exists a continuous mapping $f : X \to X$ such that $|f^{-1}[\{x\}]| = \mathfrak{c}$ for each $x \in X$. We proved that if X is a fiberable Polish space then there exists a continuous mapping $f : X \to X$ such that every Bernstein subset B of X is mapped by f onto X.

A topological space X is *productable* if there are a topological space T, $|T| = \mathfrak{c}$, and a continuous surjection from X onto $X \times T$. One can see that every productable space is fiberable. Indeed, let $g : X \to X \times T$. Then $f = \pi_X \circ g$ satisfies the condition from the definition of a fiberable space (π_X denotes the projection of $X \times T$ onto X). As there exists a Peano mapping from $\mathbb{R}$ onto $\mathbb{R}^2$, $\mathbb{R}$ is fiberable and Osipov's question is (strongly) answered in the positive.

If we do not want the topology on T to have any specific properties (e.g. if we do not want T to satisfy any separation axiom) the notions of separable space and productable space coincide.

The aim of the talk will be to present an example of a fiberable (thus also productable) metric continuum X for which there is no topological space T with nontrivial topology such that there exists a continuous mapping from X onto $X \times T$.

Thus one can make the definition of *productable* more precise requiring some properties from the topology on T. We call a space X T_i*-productable*, $i = -1, 0, 1, 2$, if there are a T_i topological space T, $|T| = \mathfrak{c}$, and a continuous surjection from X onto $X \times T$ ("T_{-1}" - means no separation property assumed). We discuss the existence of metric continua which are T_i-productable but not T_{i+1}-productable.