TWO $\mathfrak{b}$ OR NOT TWO $\mathfrak{b}$

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The bounding number $\mathfrak{b}$ is the smallest size of an unbounded family in the poset $(\omega^\omega, \leq^*)$.

In [6], the author proved that $\mathfrak{b}$ is also the smallest cardinal κ for which there exists a gap of the type (ω, κ) in the poset $(\mathcal{P}(\omega)/fin, \subseteq^*)$, whereas in [2], the authors proved that $\mathfrak{b}$ is the smallest size of a QN-space (i.e. a topological space which does not distinguish pointwise and quasinormal convergence of real functions).

The classical notions mentioned above can be idealized to $\leq^{\mathcal{I}}$-order, $\mathcal{I}$-gaps and $\mathcal{I}$-convergence, where $\mathcal{I}$ is an ideal on ω. Then, one can use these idealized notations to define an ideal version of the $\mathfrak{b}$ number:

- in [3], the authors use $\leq^{\mathcal{I}}$-order to define $\mathfrak{b}(\mathcal{I})$,
- in [1], the authors consider $\mathcal{I}$-gaps to define $\mathfrak{b}(\mathcal{I})$,
- in [5] and [4], the authors use $\mathcal{I}$-convergence to define $\mathfrak{b}(\mathcal{I})$.

In the talk, I will show what is know about these three species of idealized $\mathfrak{b}$ numbers.

References

[1] J. Brendle, D. A. Mejía, *Rothberger gaps in fragmented ideals*, Fund. Math. 227 (2014), no. 1, 35–68.

[2] L. Bukovský, I. Recław, M. Repický, *Spaces not distinguishing pointwise and quasinormal convergence of real functions*, Topology Appl. 41 (1991), no. 1-2, 25–40.

[3] B. Farkas, L. Soukup, *More on cardinal invariants of analytic P-ideals*, Comment. Math. Univ. Carolin. 50 (2009), no. 2, 281–295.

[4] R. Filipów, A. Kwela, *Yet another ideal version of the bounding number*, to appear in J. Symb. Log.

[5] R. Filipów, M. Staniszewski, *Pointwise versus equal (quasi-normal) convergence via ideals*, J. Math. Anal. Appl. 422 (2015), no. 2, 995–1006.

[6] F. Rothberger, *Sur les familles indénombrables de suites de nombres naturels et les problémes concernant la propriété C*, Proc. Cambridge Philos. Soc. 37 (1941), 109–126.

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