LET'S PLAY CHAOS

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The main goal of this lecture is introducing various kinds of chaotic points for multifunctions and later on showing their applications to the theory of infinite topological games. This issue is the part of the general thematic line considered in our team: the study of local aspects of discrete dynamical system.

Our considerations will be connected with dynamical systems created by functions or multifunctions operating in topological manifolds being compact metric space (including the unit interval).

Following the definitions in [19] and [2] we adopt the basic notions of lower- and upper distribution function which are related to asymptotic density, which was researched as a part of real analysis. The consequence of this are the notions of distributionally scrambled set and distributionally chaotic system (in the case of multifunction, we consider the Hausdorff metric).

Let $(f_{1,\infty})$ be a dynamical system consisting of functions or multifunctions. We shall say that $x_0 \in X$ is a DC1 point (distributionally chaotic point) of $(f_{1,\infty})$ if for any $\varepsilon > 0$ there exists an uncountable set S being a DS-set for $(f_{1,\infty})$ such that there are $n \in \mathbb{N}$ and a closed set $A \supset S$ fulfilling the condition

$$A \subset \zeta_1^{i \cdot n}(A) \subset B(x_0, \varepsilon)$$

for $i \in \mathbb{N}$.

The lecture will cover the following issues:

- the existence of DC1 points (connected with functions) including our results (2019) and the latest (December 2021) achievements of the Spanish-Czech team;

- problems of strategy in infinite topological games;
- application of DC1 points of multifunctions to the theory of infinite topological games.

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