Destruction of CPE-normality by deterministic sequences.

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Abstract: A number $x = 0, x_1, x_2, ...$ (say, in decimal expansion) is *normal* if the sequence $x_1, x_2, ...$ is generic for the uniform Bernoulli measure (i.i.d.) on ten symbols. In 1949 D.D. Wall proved that if $0, x_1 x_2 ...$ is a normal number, then $0, x_{i_1} x_{i_2} ...$ is also normal, whenever $i_1, i_2, ...$ is an infinite arithmetic progression. We say that *arithmetic progressions preserve normality.* This result has been later generalized by Kamae: the same holds whenever $i_1, i_2, ...$ is a *deterministic sequence* (roughly speaking has entropy zero). In other words, *deterministic sequences preserve normality.* Jointly with Adam Abrams we have show something opposite for all other measures which have completely positive entropy but are not i.i.d.: if $x_1, x_2, ...$ is λ-normal (meaning generic for a measure λ on a symbolic space), where λ has completely positive entropy but is not i.i.d., and $i_1, i_2, ...$ is a deterministic sequence then $x_{i_1}, x_{i_2}, ...$ is **never** λ-normal. More is true: in fact no sequence preserves λ-normality (not counting sequences of density 1, which trivially preserve any kind of normality).