ALMOST CONTINUOUS SIERPIŃSKI-ZYGMUND FUNCTIONS UNDER DIFFERENT SET-THEORETICAL ASSUMPTIONS

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(JOINT WORK WITH K. CIESIELSKI AND D.L. RODRÍGUEZ-VIDANES)

A function $f\colon \mathbb{R} \to \mathbb{R}$ is:

- *almost continuous* in the sense of Stallings, $f \in \mathrm{AC}$, if each open set $G \subset \mathbb{R}^2$ containing the graph of f contains also the graph of a continuous function $g\colon \mathbb{R} \to \mathbb{R}$;
- *Sierpiński-Zygmund*, $f \in \mathrm{SZ}$ (or, more generally, $f \in \mathrm{SZ(Bor)}$), provided its restriction $f \restriction M$ is discontinuous (not Borel, respectively) for any $M \subset \mathbb{R}$ of cardinality continuum.

It is known that:

(1) an example of a Sierpiński-Zygmund almost continuous function $f\colon \mathbb{R} \to \mathbb{R}$ (i.e., an $f \in \mathrm{SZ} \cap \mathrm{AC}$) cannot be constructed in ZFC;
(2) an $f \in \mathrm{SZ} \cap \mathrm{AC}$ exists under the additional set-theoretical assumption $\mathrm{cov}(\mathcal{M}) = \mathfrak{c}$, that is, that $\mathbb{R}$ cannot be covered by less than $\mathfrak{c}$-many meager sets [1].

The primary purpose of this talk is to show that the existence of an almost continuous Sierpiński-Zygmund function is consistent with ZFC plus the negation of $\mathrm{cov}(\mathcal{M}) = \mathfrak{c}$. More precisely,

(3) it is consistent with ZFC+"$\mathrm{cov}(\mathcal{M}) < \mathfrak{c}$" (follows from the assumption that $\mathrm{non}(\mathcal{N}) < \mathrm{cov}(\mathcal{N}) = \mathfrak{c}$) that $\mathrm{SZ(Bor)} \cap \mathrm{AC} \neq \emptyset$.

We also discuss, assuming either $\mathrm{cov}(\mathcal{M}) = \mathfrak{c}$ or $\mathrm{non}(\mathcal{N}) < \mathrm{cov}(\mathcal{N}) = \mathfrak{c}$, the lineability and the additivity coefficient of the class of all almost continuous Sierpiński-Zygmund functions. Several open problems will be posed in this context.

References

[1] M. Balcerzak, K. Ciesielski, and T. Natkaniec, *Sierpiński-Zygmund functions that are Darboux, almost continuous or have a perfect road*, Archive for Math. Logic **37**(1) (1997), 29–35.
[2] K.C. Ciesielski, T. Natkaniec, and D.L. Rodríguez-Vidanes, *Almost continuous Sierpiński-Zygmund functions under different set-theoretical assumptions*, submitted.

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