THE (δ)-PROPERTY FOR THE FAMILY OF BAIRE-α FUNCTIONS

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Let Ω be a perfectly normal topological space, let A be a non-empty subset of Ω and let $\mathcal{B}_\alpha(A)$ denote the space of all functions $A \to \mathbb{R}$ of Baire class $\alpha \geq 1$, where α is an ordinal number $< \omega_1$. A short and direct version of proof of the Kuratowski Extension Theorem for Baire-one functions had lead us to the generalization of this theorem to the case of extensions of Baire-α functions. This generalization allowed us to prove that $\mathcal{B}_\alpha(\Omega)$ has the so-called (δ)-*property* for linear lattices: for all $f, g \in \mathcal{B}_\alpha(\Omega)$ with $f \wedge g = 0$ there exists a Borel subset $A \subset \Omega$ of ambiguous class α with $\chi_A \cdot f = f$ and $\chi_A \cdot g = 0$, where χ_A is the characteristic function of A. The (δ)-property implies the spectral Freudenthal property and was first studied independently by Veksler and Lavrič. It was also studied in detail by Lipecki and Wójtowicz.

References

[1] O.F.K. Kalenda, J. Spurný, *Extending Baire-one functions on topological spaces*, Topology Appl. 149 (2005), no. 1-3, 195–216.

[2] B. Lavrič, *On Freudenthal's spectral theorem*, Indag. Math. 48 (1986), 411-421.

[3] Z. Lipecki, *On binary-type approximations in vector lattices*, Arch. Math. (Basel) 62 (1994), 545–553.

[4] W. A. J. Luxemburg, A. C. Zaanen *Riesz spaces I*, North-Holland, Amsterdam and London, 1966.

[5] H. H. Schaefer, *Banach lattices and positive operators*, Berlin-Heidelberg-New York 1974.

[6] H. R. Shatery, J. Zafarani, *The equality between Borel and Baire classes*, Real Anal. Exch. 30 (2004/2005), 373–384.

[7] A. I. Veksler, *Projection properties of linear lattices Freudenthal's theorem* (in Russian), Math. Nachr. 74 (1976), 7-25.

[8] M. Wójtowicz, *On a weak Freudenthal's spectral theorem*, CMUC 33,4 (1992), 631-643.

[9] A. C. Zaanen, *Examples of orthomorphisms*, J. App. Th. 13, (1975), 192–204.

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