INSPIRATIONS IN REAL ANALYSIS

BOOK OF ABSTRACTS

BĘDLEWO, 3–8.04.2022

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Banach Center, Institute of Mathematics PAS

Lodz University of Technology

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The conference *Inspirations in Real Analysis* is addressed especially to people interested in real analysis, set theory, topology, dynamical systems and also other topics which are close to the ones just mentioned. These areas of mathematics are interrelated but conferences usually separate them from one another, hence the idea to bring together researchers sharing an interest in such fields of mathematics.

The aim of the meeting is to exchange experiences and ideas among scientists to give rise to new research trends not only in real analysis (as the name of the conference suggests), but also in set theory, topology and dynamical systems. Our invitation for contributing a plenary talk has been accepted by professors:

- Emma D'Aniello (Italy, Università degli Studi della Campania Luigi Vanvitelli)
- Udayan Darji (USA, University of Louisville)
- Tomasz Downarowicz (Wrocław University of Science and Technology)
- Márton Elekes (Hungary, Eötvös Loránd University, Budapest; Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences)
- Rafał Filipów (University of Gdańsk)
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Authomatic continuity of measurable homomorphisms between topological groups

Taras Banakh (Ivan Franko National University of Lviv)

Theorem.

1. *Every Haar-measurable homomorphism from a locally compact group to any topological group is continuous.*
2. *Every BP-measurable homomorphism from any ω-narrow Čech-complete group to any topological group is continuous.*
3. *Every Borel homomorphism from any Čech complete group to any topological group is continuous.*

Center of distances and central Cantor sets -part II

Michał Banakiewicz (Szczecin University of Technology),
Artur Bartoszewicz (University of Lodz),
Małgorzata Filipczak (University of Lodz),
Franciszek Prus-Wiśniowski (University of Szczecin)

Any central Cantor set C is an achievement set of some fast convergent sequence (a_n). It is known that the center of distances of C contains all numbers a_n. We observe when the center of distances of C is larger then the minimal one.

The Freese–Nation property

Judyta Bąk (Jan Kochanowski University in Kielce)

The Freese–Nation property was considered for lattices. We introduce this property for topological spaces and indicate some classes of topological spaces which have this property. We show that these spaces can be represented as the limit of an inverse system satisfying some conditions.

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Variational principles of a sequence of maps

Andrzej Biś (University of Lodz)

The authors of [1] have applied Convex Analysis approach to get a general variational principle. In a case of a sequence of continuous self-maps, there are several entropy-like quantities that lead to several pressure functions. In the talk, I intend to present the applications of the results of [1] to get the variational principles of a sequence of continuous self-maps. A presentation is based on a joint work with A. Marczuk.

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On kneading determinat in two variables

Jozef Bobok (Czech Technical University in Prague)

Let f be a unimodal interval map given for $a > 0, b > 1$ by the formula

$$f_{a,b}(x) := \begin{cases} ax + \frac{a+b-ab}{b}, & x \in [0, 1 - \frac{1}{b}], \\ b - bx, & x \in [1 - \frac{1}{b}, 1]. \end{cases}$$

We assume the values a, b to define the map $f_{a,b}$ topologically mixing. We study the properties of the kneading determinant of f as a function of two variables.

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Chaos and hyperbolic properties for linear operators on L^p spaces

Emma D'Aniello
(Universita' degli Studi della Campania "Luigi Vanvitelli")

We investigate notions like transitivity, Li-Yorke chaos coming from topological dynamics in the linear setting, and new notions originally defined in a non-linear framework like hyperbolicity, the shadowing property and expansivity. We focus in particular on the class of composition operators on arbitrary measure spaces. This class includes the weighted shifts as a special case.

Local Entropy and Descriptive Complexity

Udayan Darji (University of Louisville)
Felipe Garcia-Ramos (University of British Columbia)

We investigate local entropy theory, particularly the properties of having uniform positive entropy and completely positive entropy, from a descriptive set-theoretic point of view. We show natural classes of dynamical systems which form Borel sets as well as coanalytic non-Borel sets. In particular, we show that the class of systems with uniform positive entropy and the class of systems with the shadowing property having completely positive entropy is Borel. Meanwhile, the class of mixing systems on a Cantor space is coanalytic but not Borel, and the class of systems on the interval and other orientable manifolds with CPE are coanalytic complete.

Measures as graph limits

Martin Doležal
(Institute of Mathematics of the Czech Academy of Sciences)

We investigate so called s-convergence, which is one of the many convergence notions of sequences of graphs, recently introduced by Kunszenti-Kovács, Lovász, and Szegedy in [2]. We provide an alternative approach to s-convergence. The original definition is based on the convergence of certain compact sets, called k-shapes, of k-by-k matrices. We show that this is equivalent to the convergence of certain compact sets of Borel probability measures.

This talk is based on the paper [1].

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Destruction of CPE-normality by deterministic sequences

Tomasz Downarowicz
(Wrocław University of Science and Technology)

A number $x = 0, x_1, x_2, ...$ (say, in decimal expansion) is *normal* if the sequence $x_1, x_2, ...$ is generic for the uniform Bernoulli measure (i.i.d.) on ten symbols. In 1949 D.D. Wall proved that if $0, x_1x_2...$ is a normal number, then $0, x_{i_1}x_{i_2}...$ is also normal, whenever $i_1, i_2, ...$ is an infinite arithmetic progression. We say that *arithmetic progressions preserve normality.* This result has been later generalized by Kamae: the same holds whenever $i_1, i_2, ...$ is a *deterministic sequence* (roughly speaking has entropy zero). In other words, *deterministic sequences preserve normality.* Jointly with Adam Abrams we have show something opposite for all other measures which have completely positive entropy but are not i.i.d.: if $x_1, x_2, ...$ is λ-normal (meaning generic for a measure λ on a symbolic space), where λ has completely positive entropy but is not i.i.d., and $i_1, i_2, ...$ is a deterministic sequence then $x_{i_1}, x_{i_2}, ...$ is **never** λ-normal. More is true: in fact no sequence preserves λ-normality (not counting sequences of density 1, which trivially preserve any kind of normality).

On various notions of universally Baire sets and their applications to Haar meagreness

Márton Elekes
(Eötvös Loránd University, Budapest;
Alfréd Rényi Institute of Mathematics;
Hungarian Academy of Sciences)

Universally Baire sets play a crucial role in set theory, and they are also very interesting from the point of view of descriptive set theory. However, there are at least a dozen different definitions, and many of these are indeed non-equivalent (at least consistently). The goal of this talk is to clarify this situation, and also to give applications in the theory of so called Haar meagre sets. Joint work with Máté Pálfy.

Center of distances and central Cantor sets

Michał Banakiewicz (Szczecin University of Technology),
Artur Bartoszewicz (University of Lodz),
Małgorzata Filipczak (University of Lodz),
Franciszek Prus-Wiśniowski (University of Szczecin)

We study a recently discovered metric invariant - the center of distances - that is particularly useful in investigating achievability of sets on the real line. We concentrate on the question which central Cantor sets have the minimal possible center of distances and which have not.

Two $\mathfrak{b}$ or not two $\mathfrak{b}$

Rafał Filipów (University of Gdańsk)

The bounding number $\mathfrak{b}$ is the smallest size of an unbounded family in the poset $(\omega^\omega, \leq^*)$.

In [6], the author proved that $\mathfrak{b}$ is also the smallest cardinal κ for which there exists a gap of the type (ω, κ) in the poset $(\mathcal{P}(\omega)/fin, \subseteq^*)$, whereas in [2], the authors proved that $\mathfrak{b}$ is the smallest size of a QN-space (i.e. a topological space which does not distinguish pointwise and quasinormal convergence of real functions).

The classical notions mentioned above can be idealized to $\leq^\mathcal{I}$-order, $\mathcal{I}$-gaps and $\mathcal{I}$-convergence, where $\mathcal{I}$ is an ideal on ω. Then, one can use these idealized notations to define an ideal version of the $\mathfrak{b}$ number:

- in [3], the authors use $\leq^\mathcal{I}$-order to define $\mathfrak{b}(\mathcal{I})$,
- in [1], the authors consider $\mathcal{I}$-gaps to define $\mathfrak{b}(\mathcal{I})$,
- in [5] and [4], the authors use $\mathcal{I}$-convergence to define $\mathfrak{b}(\mathcal{I})$.

In the talk, I will show what is know about these three species of idealized $\mathfrak{b}$ numbers.

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Gurariĭ operators are generic

Joanna Garbulińska-Węgrzyn
(Jan Kochanowski University in Kielce)

An operator $U : V \to W$ between Banach spaces is defined to be *universal* if for every operator $T : X \to Y$ with $\|T\| \leq \|U\|$, there exist linear isometric embeddings $i : X \to V$, $j : Y \to W$ such that $U \circ i = j \circ T$.

In [2] was constructed a universal operator $\Omega : \mathbb{G} \to \mathbb{G}$, where $\mathbb{G}$ denotes the Gurariĭ space. More precisely, it was introduced the notion of a Gurariĭ operator (which is an operator counterpart of the notion of a Gurariĭ space) and was presented a construction of a Gurariĭ operator (as the Fraisse limit in a suitable category). Moreover, it was proven that every Gurariĭ operator is universal.

An operator $G : X \to Y$ between Banach spaces is called *Gurariĭ* if G is nonexpansive and for any $\varepsilon > 0$, any nonexpansive operator $T : A \to B$ between finite-dimensional Banach spaces, any Banach subspaces $A_0 \subseteq A$, $B_0 \subseteq B$ with $T[A_0] \subseteq B_0$, and any isometric embeddings $i_0 : A_0 \subseteq X$, $j_0 : B_0 \subseteq Y$ with $G \circ i_0 = j_0 \circ T\restriction_{A_0}$, there exist ε-isometric embeddings $i : A \to X$ and $j : B \to Y$ such that $i\restriction_{A_0} = i_0$, $j\restriction_{B_0} = j_0$ and $G \circ i = j \circ T$.

In this talk we will present several characterizations of Gurariĭ operators. The main result shows that the Gurariĭ operators form a dense G_δ-set in the space

$B(\mathbb{G})$ of all nonexpansive operators on the Gurariĭ space $\mathbb{G}$, endowed with the strong operator topology. This implies that universal operators on $\mathbb{G}$ form a residual set in $B(\mathbb{G})$.

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On n-saturated closed graphs

Szymon Głąb (Lodz University of Technology),
Przemysław Gordinowicz (Lodz University of Technology)

Geschke proved that there is clopen graph on 2^ω which is 3-saturated, but the clopen graphs on 2^ω do not even have infinite subgraphs that are 4-saturated; however there is F_σ graph that is ω_1-saturated. It turns out that there is no closed graph on 2^ω which is ω-saturated. We complete this picture by proving that for every $n \in \mathbb{N}$ there is an n-saturated closed graph on the Cantor space 2^ω. The key lemma is based on probabilistic argument. The final construction is an inverse limit of finite graphs. This is a joint work with Przemysław Gordinowicz: https://arxiv.org/abs/2201.10932

Semiattractors of multifunctions on product spaces and generalized iterated function systems

Grzegorz Guzik (AGH University of Science and Technology)

We introduce the notion of semiattractor for lower semicontinuous multifunctions defined on a finite product of an arbitrary metric space. Then we obtain semiattractors (semifractals) for a large class of generalized iterated function systems (GIFSs). The theory of semiattractors was developed by A. Lasota and J. Myjak in the context of a single multifunction associated with an iterated function system (IFS), as well as supports of invariant measures for some transition Markov operators induced by IFSs with probability. Our results are the counterpart of these. We prove some fundamental properties of semiattractors, give the explicit construction, some simple criteria of existence and also the theorem on approximation of the semiattractor of a countable GIFS by semiattractors of its finite subsystems. We use the apparatus of topological (Kuratowski's) limits.

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Around the continuity of the operator of ”center of distances”

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Sebastian Lindner (University of Lodz),
Franciszek Prus-Wiśniowski (University of Szczecin)

The notion of a center of distances is an interesting invariant of a metric space. For a given metric space X with a distance ρ *the center of distances* is defined by W. Bielas, S. Plewik and M. Walczyńska as follows:

$$S(X) := \{\alpha : \forall_{x\in X} \ \ \exists_{y\in X} \ \ \rho(x,y) = \alpha\}.$$

We consider this operator defined on the family $K([0,1])$ of all nonempty compact subsets of the interval $[0,1]$ equipped with the Hausdorff metric H. We show that $S : (K([0,1]), H) \to (K([0,1]), H)$ is not continuous nor open, has a dense set of continuity points and has a dense set of discontinuity points. We characterize the set of continuity points of S, and prove that S is upper semicontinuous at any $A \in K([0,1])$, so it is of the first Borel class.

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Metric compactness criteria involving sequences of mappings and a proof of the Ascoli–Arzelà theorem with the use of Bernstein polynomials

Jacek Jachymski (Lodz University of Technology)

We establish inter alia a compactness criterion in metric spaces involving a sequence of completely continuous mappings, which is continuously convergent, in the sense of H. Hahn (see, e.g., [6, p. 197]), to the identity mapping. For Banach spaces, the linear version of that result coincides with the compactness theorem due to S. Mazur, which was first mentioned, without a proof, in Banach's French monograph [1, p. 237]. We also present probably a new proof of the Ascoli–Arzelà theorem, in which we use the above compactness criterion applied to the sequence of Bernstein operators. Let us note that in classical proofs of the Ascoli–Arzelà theorem either a finite ε-net for a suitable family of functions is constructed (see, e.g., [3] or [8, p. 394]), or a diagonalization argument is used as done, e.g., in [5, p. 154]. There are also other lesser-known approaches: the proof given by Ullrich [9] is based on the Tychonoff compactness theorem; Nagy [7] presented a functional analytic proof with the help of the Banach–Alaoglu theorem; Beer [2] derived the result from the Zarankiewicz

compactness theorem for sequences of closed sets in a separable metric space; Hanche-Olsen and Holden [4] proved the theorem via a clever simple lemma on metric compactness, which, however, is completely different from our criterion. At last, recently, Wójtowicz [10] provided yet another proof that uses the Stone–Čech compactification technique.

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On Δ-spaces X and their characterization in terms of spaces $C_p(X)$

Jerzy Kąkol (Adam Mickiewicz University in Poznań)

Reed (see [6], [4]) studied those uncountable subsets D (under the name Δ-sets) of the reals $\mathbb{R}$ (with the natural topology) having the following property:

For any decreasing sequence $(H_n)_n$ of subsets of D with $\bigcap_n H_n = \emptyset$ there is a sequence $(V_n)_n$ of G_δ-subsets of D such that $H_n \subset V_n$, $n \in \mathbb{N}$, and $\bigcap_n V_n = \emptyset$. Research about Δ-spaces is strictly connected with a study of $\mathbb{Q}$-sets, one of the most mysterious objects in $\mathbb{R}$. In [2] the concept of a Δ-set has been extended to arbitrary topological spaces: A topological space X is called a *Δ-space* if for every decreasing sequence $(D_n)_n$ of subsets of X with $\bigcap_n D_n = \emptyset$, there is a decreasing sequence $(V_n)_n$ of open subsets of X, $D_n \subset V_n$ for every $n \in N$ and $\bigcap_n V_n = \emptyset$. In [2] we proved that X is a Δ-space if and only the dual of $C_p(X)$ endowed with the topology of the uniform convergence on $C_p(X)$-bounded sets carries the finest locally convex topology. This analytic approach provided several new results about Δ-sets and Δ-spaces [2], [3], [5]. Some alternative characterization was also presented in [1]. Applications for Banach spaces $C(K)$ and spaces $C_p(K)$ are provided.

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The character of convergence of the Cauchy product

Adam Krupowies (University of Szczecin)
Franciszek Prus-Wiśniowski (University of Szczecin)

In general, the Cauchy product of an absolutely convergent series and a conditionally convergent one might converge absolutely. In our talk, we provide an easy and quite general method for construction of such pairs of series, a method that is not related to the classic Pringsheim's example. Moreover, we observe that when only pairs of alternating series are considered if one of them is absolutely convergent then the character of convergence of their Cauchy product is exactly the same as the character of convergence of the second factor. We complete the remarks with a new and astonishingly short proof of the Voss Theorem on Cauchy product.

On P-spaces

Andrzej Kucharski (University of Silesia in Katowice)
(joint work with Wojciech Bielas (University of Silesia in Katowice)
and Szymon Plewik (University of Silesia in Katowice))

We discuss the connection between inverse limits of height ω_1 of discrete topological spaces and P-spaces. Results are concentrated on dimensional types of some P-spaces. The name "P-space" was used by L. Gillman and M. Henriksen [2]. If a space X is completely regular and every countable intersection of open sets of X is open, then X is called *P-space*. If X is topologically embedded in Y, then the dimensional type of X is less or equal to the dimensional type of Y (see [3] or [4]).

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Orbit pseudometrics and a universality property of the Gromov-Hausdorff distance

Ondřej Kurka
(Institute of Mathematics of the Czech Academy of Sciences)

In the talk, we consider the notion of Borel reducibility between pseudometrics on standard Borel spaces introduced and studied recently by Cúth, Doucha and Kurka, as well as the notion of an orbit pseudometric, a continuous version of the notion of an orbit equivalence relation. It is well known that the relation of isometry of Polish metric spaces is bireducible with a universal orbit equivalence relation. We prove a version of this result for pseudometrics, showing that the Gromov-Hausdorff distance of Polish metric spaces is bireducible with a universal element in a certain class of orbit pseudometrics.

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Large vector spaces of bounded sequences

Paolo Leonetti (Bocconi University)

It has been recently proved in [2] that the set S of bounded non-convergent real sequences contains, except for zero, a dense vector space V with dimension continuum. We partition S into three sets $\{S_1, S_2, S_3\}$ and show that each of them has the same property. Then, we study the same question for V closed (in place of dense) and conclude with several open questions.

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Rate of convergence in the chaos game

Krzysztof Leśniak (Nicolaus Copernicus University in Toruń)
(joint work with Nina Snigireva (Dublin City University)
and Filip Strobin (Lodz University of Technology))

It is known that an attractor of a contractive iterated function system is the omega-limit of the orbit that is driven by a disjunctive sequence (i.e., a sequence of symbols, which contains all possible finite words). In particular, this convergence holds with probability 1, when the orbit is driven by a sequence generated by a chain with complete connections with positively minorized transition probabilities, the most simple case being a Bernoulli scheme. Very recently, Bárány, Jurga and Kolossváry have established the rate of convergence of the probabilistic chaos game in terms of the box dimension, cf. [1]. We will present what happens to the rate of convergence when a disjunctive chaos game is considered instead of the probabilistic one, cf. [2].

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Free group of Hamel functions

Mateusz Lichman (Lodz University of Technology)

Given a function $f : \mathbb{R} \to \mathbb{R}$ we say that f is a Hamel function if it is a $\mathbb{Q}$-basis of the linear space $\mathbb{R}^2$. In paper [1] Authors provided constructions of some Hamel functions with additional properties. Among others, they have constructed a Hamel autobijection of $\mathbb{R}$. Developing their ideas we have constructed a free group of $\mathfrak{c}$ generators of Hamel autobijections of $\mathbb{R}$. During my talk I will cite the basic tools which we used, sketch the construction of such a free group and sum up with some open questions. This is a joint work with M. Pawlikowski (Lodz University of Technology), Sz. Smolarek (Lodz University of Technology) and J. Swaczyna (Lodz University of Technology).

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Generalization of Mycielski theorem

Marcin Michalski (Wrocław University of Science and Technology)

Let us recall the following two-dimensional version of the classical theorem of Mycielski.

Theorem (Mycielski [2])**.** *For every comeager or conull set* $X \subseteq [0,1]^2$ *there exists a perfect set* $P \subseteq [0,1]$ *such that* $P \times P \subseteq X \cup \Delta$*.*

We will consider strengthening of this theorem by replacing a perfect square with a square or a rectangle $A \times B$ of bodies of some types of trees satisfying $A \subseteq B$. In particular we will examine possible generalizations for Miller, Laver, Silver, splitting and uniformly perfect trees.

The results were obtained together with Robert Rałowski (Wrocław University of Science and Technology) and Szymon Żeberski (Wrocław University of Science and Technology) and part of them can be found in [1].

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A characterization of the fuzzy fractals generated by an orbital fuzzy iterated function system

Radu Miculescu (Transilvania University of Braşov)

Orbital fuzzy iterated function systems are obtained as a combination of the concepts of iterated fuzzy set system and orbital iterated function system. It turns out that, for such a system, the corresponding fuzzy operator is weakly Picard, its fixed points being called fuzzy fractals. In this paper we present a structure result concerning fuzzy fractals associated to an orbital fuzzy iterated function system by proving that such an object is perfectly determined by the action of the initial term of the Picard iteration sequence on the closure of the orbits of certain elements.

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Real Analytic Inspiration – from Continuous Images of Bernstein Sets to (non-)Productable Continua

Jacek Cichoń, **Michał Morayne**, Robert Rałowski
(Wrocław University of Science and Technology)

A. Osipov asked if for every Bernstein set $B \subseteq \mathbb{R}$ the real line can be covered by a countable family of sets each of which is a continuous image of B.

A topological space X is *fiberable* if there exists a continuous mapping $f : X \to X$ such that $|f^{-1}[\{x\}]| = \mathfrak{c}$ for each $x \in X$. We proved that if X is a fiberable Polish space then there exists a continuous mapping $f : X \to X$ such that every Bernstein subset B of X is mapped by f onto X.

A topological space X is *productable* if there are a topological space T, $|T| = \mathfrak{c}$, and a continuous surjection from X onto $X \times T$. One can see that every productable space is fiberable. Indeed, let $g : X \to X \times T$. Then $f = \pi_X \circ g$ satisfies the condition from the definition of a fiberable space (π_X denotes the projection of $X \times T$ onto X). As there exists a Peano mapping from $\mathbb{R}$ onto $\mathbb{R}^2$, $\mathbb{R}$ is fiberable and Osipov's question is (strongly) answered in the positive.

If we do not want the topology on T to have any specific properties (e.g. if we do not want T to satisfy any separation axiom) the notions of separable space and productable space coincide.

The aim of the talk will be to present an example of a fiberable (thus also productable) metric continuum X for which there is no topological space T with <u>nontrivial</u> topology such that there exists a continuous mapping from X onto $X \times T$.

Thus one can make the definition of *productable* more precise requiring some properties from the topology on T. We call a space X T_i*-productable*, $i = -1, 0, 1, 2$, if there are a T_i topological space T, $|T| = \mathfrak{c}$, and a continuous surjection from X onto $X \times T$ ("T_{-1}" - means no separation property assumed). We discuss the existence of metric continua which are T_i-productable but not T_{i+1}-productable.

Almost continuous Sierpiński-Zygmund functions under different set-theoretical assumptions

Tomasz Natkaniec (University of Gdańsk)
(joint work with K.C. Ciesielski (West Virginia University)
and D.L. Rodríguez-Vidanes (Complutense University of Madrid))

A function $f\colon \mathbb{R} \to \mathbb{R}$ is:

- *almost continuous* in the sense of Stallings, $f \in$ AC, if each open set $G \subset \mathbb{R}^2$ containing the graph of f contains also the graph of a continuous function $g\colon \mathbb{R} \to \mathbb{R}$;
- *Sierpiński-Zygmund*, $f \in$ SZ (or, more generally, $f \in$ SZ(Bor)), provided its restriction $f \restriction M$ is discontinuous (not Borel, respectively) for any $M \subset \mathbb{R}$ of cardinality continuum.

It is known that:

1. an example of a Sierpiński-Zygmund almost continuous function $f\colon \mathbb{R} \to \mathbb{R}$ (i.e., an $f \in \mathrm{SZ} \cap \mathrm{AC}$) cannot be constructed in ZFC;
2. an $f \in \mathrm{SZ} \cap \mathrm{AC}$ exists under the additional set-theoretical assumption $\mathrm{cov}(\mathcal{M}) = \mathfrak{c}$, that is, that $\mathbb{R}$ cannot be covered by less than $\mathfrak{c}$-many meager sets [1].

The primary purpose of this talk is to show that the existence of an almost continuous Sierpiński-Zygmund function is consistent with ZFC plus the negation of $\mathrm{cov}(\mathcal{M}) = \mathfrak{c}$. More precisely,

(3) it is consistent with ZFC+"$\text{cov}(\mathcal{M}) < \mathfrak{c}$" (follows from the assumption that $\text{non}(\mathcal{N}) < \text{cov}(\mathcal{N}) = \mathfrak{c}$) that $\text{SZ}(\text{Bor}) \cap \text{AC} \neq \emptyset$.

We also discuss, assuming either $\text{cov}(\mathcal{M}) = \mathfrak{c}$ or $\text{non}(\mathcal{N}) < \text{cov}(\mathcal{N}) = \mathfrak{c}$, the lineability and the additivity coefficient of the class of all almost continuous Sierpiński-Zygmund functions. Several open problems will be posed in this context.

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Pointwise attractors which are not strict

Magdalena Nowak (Jan Kochanowski University in Kielce)

We deal with the finite family $\mathcal{F}$ of continuous maps on the normal Hausdorff space. Each nonempty compact subset A of such space is called a strict attractor if it has an open neighbourhood U such that $A = \lim_{n\to\infty} \mathcal{F}^n(S)$ for every nonempty compact $S \subset U$. Every strict attractor is a pointwise attractor, which means that the set $\{x \in X; \lim_{n\to\infty} \mathcal{F}^n(x) = A\}$ contains A in its interior.

We present a class of examples of pointwise attractors which are not strict - from the finite set to the Sierpiński gasket.

On the algebraic difference of special Cantor sets

Piotr Nowakowski (University of Lodz)

We investigate some self-similar Cantor sets $C(l, r, p)$, which we call special Cantor sets (or in short S-Cantor sets), generated by numbers $l, r, p \in \mathbb{N}$, $l + r < p$. We give a full characterization of the set $C(l_1, r_1, p) - C(l_2, r_2, p)$ which can take one of the form: the interval $[-1, 1]$, a Cantor set, an L-Cantorval, an R-Cantorval or an M-Cantorval.

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On some ideals defined by density topologies on the real line and in the Cantor space.

Andrzej Nowik (University of Gdańsk)

The condition (in fact, an ideal) (a) was first introduced in [1] and investigated in many papers ([5], [3], [4], [N]). We can define it using the density topology τ_d, namely $(a) = \{A \subseteq \mathbb{R} \colon \forall_{U \in \tau_d \setminus \{\emptyset\}} \exists_{W \in \tau_e} U \cap W \neq \emptyset \wedge U \cap W \cap A = \emptyset\}$ (where τ_e is the standard topology on $\mathbb{R}$).

- It is known that $(a) \subsetneq \mathsf{ND} \cap \mathcal{N}$, where ND is the collection of nowhere dense sets and $\mathcal{N}$ is the sigma ideal of Lebesgue null sets. We prove that in the case of category density topology (see [6]) instead the density topology we obtain the ideal ND.
- It is known that (a) is equal to intersection of the collections of nowhere dense sets for all topologies between τ_e and τ_d. We prove that (a) cannot be represented as a finite intersection of the collections of nowhere dense sets for some topologies between τ_e and τ_d.
- We consider generalized version of the collection (a) for another topologies and collections and we give a construction of a G_δ set which is a counterexample to the inclusion $(a)(\tau_e, \mathcal{B}_{\mathrm{EL}}) \subseteq (a)(\tau_e, \tau_{\mathrm{EL}})$ (where τ_{EL} and $\mathcal{B}_{\mathrm{EL}}$ denote the Ellentuck topology and its standard base, respectively).

We also discuss an application of the notion of Marczewski - Burstin representability here, and we consider the another collection (a') and the problem posed in the article [2] whether the ideal generated by (a') is equal to (a). Several another open problems will be posed in this context.

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The Hutchinson-Barnsley theory for generalized iterated function systems: a dynamical approach

Elismar R. Oliveira (Federal University of Rio Grande do Sul)

The study of generalized iterated function systems (GIFS) was introduced in 2008 by Mihail and Miculescu [1]. We provide a dynamical approach to study those systems as the limit of the Hutchinson-Barnsley theory for infinite iterated function systems (IIFS) which was developed by many authors in the last years. In this talk we will explain this technique along with the main ideas used in the proofs.

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On typical properties of Lebesgue measure preserving maps in dimension one

Piotr Oprocha (AGH University of Science and Technology)
(joint work with Jozef Bobok (Czech Technical University in Prague),
Jernej Činč (University of Ostrava)
and Serge Troubetzkoy (Aix-Marseille University)

In this talk I will discuss selected properties of generic continuous maps of the interval and circle which preserve the Lebesgue measure. I will focus on a few natural properties such as entropy, structure of periodic points, mixing properties, shadowing properties, etc. I will also highlight properties of generic maps compared to other possible dynamical behaviours within maps preserving Lebesgue measure. If time permits, I will present consequences of obtained results for interval maps (not necessarily preserving Lebesgue measure) and two-dimensional dynamics.

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LET'S PLAY CHAOS

Ryszard J. Pawlak (University of Lodz)

The main goal of this lecture is introducing various kinds of chaotic points for multifunctions and later on showing their applications to the theory of infinite topological games. This issue is the part of the general thematic line considered in our team: the study of local aspects of discrete dynamical system.

Our considerations will be connected with dynamical systems created by functions or multifunctions operating in topological manifolds being compact metric space (including the unit interval).

Following the definitions in [6] and [1] we adopt the basic notions of lower- and upper distribution function which are related to asymptotic density, which was researched as a part of real analysis. The consequence of this are the notions of distributionally scrambled set and distributionally chaotic system (in the case of multifunction, we consider the Hausdorff metric).

Let $(f_{1,\infty})$ be a dynamical system consisting of functions or multifunctions. We shall say that $x_0 \in X$ is a DC1 point (distributionally chaotic point) of $(f_{1,\infty})$ if for any $\varepsilon > 0$ there exists an uncountable set S being a DS-set for $(f_{1,\infty})$ such that there are $n \in \mathbb{N}$ and a closed set $A \supset S$ fulfilling the condition

$$A \subset \zeta_1^{i \cdot n}(A) \subset B(x_0, \varepsilon)$$

for $i \in \mathbb{N}$.

The lecture will cover the following issues:

- the existence of DC1 points (connected with functions) including our results (2019) and the latest (December 2021) achievements of the Spanish-Czech team;

- problems of strategy in infinite topological games;
- application of DC1 points of multifunctions to the theory of infinite topological games.

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An inverse Fraïssé limit for finite posets and duality for posets and lattices.

Michał Pawlikowski (Lodz University of Technology)

We study the inverse limit for finite posets. Using duality for finite posets and lattices, we study connections between Fraïssé inverse sequences and inverse limit for posets and lattices. This is a join work with Szymon Głąb (Lodz University of Technology). The early version of the discussed results can be found in the preprint published on arXiV [1].

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On $\boldsymbol{T_1}$- and $\boldsymbol{T_2}$-productable compact spaces

Michał Morayne
(Wrocław University of Science and Technology),
Robert Rałowski
(Wrocław University of Science and Technology)

We prove that if there exists a continuous surjection from a metric compact space X onto a product $X \times T$ where T is a T_1 second countable topological space which has the cardinality of the continuum then there exists a surjection from X onto the product $X \times [0, 1]$ where the interval $[0, 1]$ is equipped with the usual Euclidean topology.

Some results on distributionally chaotic points

Lenka Rucká (Silesian University in Opava),
Francisco Balibrea (University of Murcia)

The distributionally chaotic point (DC point for short), is introduced in [1] as a point whose arbitrarily small neighbourhood contains an uncountable distributionally chaotic set, which is bounded by a special envelope. In this talk we extend the result from [1] and we show, that for continuous interval maps, positive topological entropy implies existence of uncountably many DC points. Also we show that this result cannot be extended to higher class of maps, particularly to continuous triangular maps of the square.

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The (δ)-property for the family of Baire-α functions

Waldemar Sieg (Kazimierz Wielki University in Bydgoszcz)

Let Ω be a perfectly normal topological space, let A be a non-empty subset of Ω and let $\mathcal{B}_\alpha(A)$ denote the space of all functions $A \to \mathbb{R}$ of Baire class $\alpha \geq 1$, where α is an ordinal number $< \omega_1$. A short and direct version of proof of the Kuratowski Extension Theorem for Baire-one functions had lead us to the generalization of this theorem to the case of extensions of Baire-α functions. This generalization allowed us to prove that $\mathcal{B}_\alpha(\Omega)$ has the so-called (δ)-*property* for linear lattices: for all $f, g \in \mathcal{B}_\alpha(\Omega)$ with $f \wedge g = 0$ there exists a Borel subset $A \subset \Omega$ of ambiguous class α with $\chi_A \cdot f = f$ and $\chi_A \cdot g = 0$, where χ_A is the characteristic function of A. The (δ)-property implies the spectral Freudenthal property and was first studied independently by Veksler and Lavrič. It was also studied in detail by Lipecki and Wójtowicz.

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Iterated function systems enriched with isometries and transition phenomena

Nina Snigireva (Dublin City University)

An iterated function system (IFS) can be enriched with an isometry in such a way that the resulting fractal set has prescribed symmetry. Such a system is an example of a noncontractive IFS. We will discuss the Lasota-Myjak theory of semiattractors and describe how it can be used to explain the behaviour of IFSs enriched with isometries. We will then show that such IFSs arise naturally in the study of a one-parameter family of IFSs. In particular, they occur at the threshold parameter between contractivity and expansion of the one-parameter family of IFSs.

(This is joint work with Krzysztof Leśniak (Nicolaus Copernicus University in Toruń), Filip Strobin (Lodz University of Technology) and A. Vince (University of Florida))

Fuzzy-set approach to invariant idempotent measures

Filip Strobin (Lodz University of Technology)
Rudnei D. da Cunha (Federal University of Rio Grande do Sul)
Eismar Oliveira (Federal University of Rio Grande do Sul)

During the talk I will present a new approach to the Hutchinson-Barnsley theory for idempotent measures first presented in [1]. The main feature developed here is a metrization of the space of idempotent measures using the embedding of the space of idempotent measures to the space of fuzzy sets. The metric obtained induces a topology stronger than the canonical pointwise convergence topology. A key result is the existence of a bijection between idempotent measures and fuzzy sets and a conjugation between the Markov operator of an IFS on idempotent measures and the fuzzy fractal operator of the associated Fuzzy IFS. This allows to prove that the Markov operator for idempotent measures is a contraction w.r.t. the induced metric and, from this, to obtain the existence of invariant idempotent measure.

References

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Getting continuity of coordinate functionals related to filter Schauder basis

Jarosław Swaczyna (Lodz University of Technology)

Given a filter of subsets of natural numbers $\mathcal{F}$ we say that a sequence (x_n) is $\mathcal{F}$-convergent to x if for every $\varepsilon > 0$ condition $\{n \in \mathbb{N} : d(x_n, x) < \varepsilon\} \in \mathcal{F}$ holds. We may use this notion to generalize the idea of Schauder basis, namely we say that a sequence (e_n) is $\mathcal{F}$-basis if for every $x \in X$ there exists a unique sequence of scalars (α_n) s.t. $\sum_{n,\mathcal{F}} \alpha_n e_n = x$, which means that the sequence of partial sums is $\mathcal{F}$ convergent to x. Once such a notion is introduced it is natural to ask whenever a corresponding coordinate functionals are continuous. Such a question was posted by V. Kadets during the 4th conference Integration, Vector Measures, and Related Topics held in 2011 in Murcia. Surprisingly, there is an obstacle related to the lack of uniform boundedness of functionals related to $\mathcal{F}$ basis, due to which we can not find a proof of continuity analogous to the classical case. During my talk I will discuss the problem and provide the proof of continuity of considered functionals. This is joint work with Tomasz Kania and Noe de Rancourt.

References

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On weakly Świątkowski functions

Małgorzata Terepeta (Lodz University of Technology)

In 2020 in the paper [1] the following definition was introduced: we say that f satisfies the weak Świątkowski condition (or is weakly Świątkowski) if for all $x_1 \neq x_2$ with $f(x_1) < f(x_2)$ there is a point $x \in I(x_1, x_2)$ such that $f(x_1) < f(x) < f(x_2)$.

This definition is a modification of Świątkowski condition ([2]) in which point x mentioned above has to be a point of continuity of f.

In the talk we will examine some properties of weakly Świątkowski functions.

All results are obtained together with Małgorzata Filipczak (University of Lodz) and Artur Bartoszewicz (University of Lodz).

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Different kinds of density ideals

Jacek Tryba (University of Gdańsk)

We consider several kinds of ideals described by some densities. We present connections between Erdős-Ulam, density, matrix summability and generalized density ideals, compare these classes of ideals and show that a certain inaccuracy in Farah's definition of density ideals leads to Farah's characterization when density ideals are Erdős-Ulam ideals being incorrect.

Convergence in ball spaces setting

Filip Turoboś (Lodz University of Technology)
joint results with Piotr Nowakowski (University of Lodz)

The notion of ball spaces, that is, a pair of a set and a nonempty family of some of its nonempty subsets, first appeared in the paper [1]. The purpose of this talk is to introduce a notion of ball convergence. We shall discuss its connection to the classical notions of convergence in topological spaces and semimetric spaces as well as some difficulties that arise while working with ball convergence.

References

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On cliquish functions in algebrability terms

Gertruda Ivanova (Pomeranian University in Słupsk),
Renata Wiertelak (University of Lodz)

Using the notions of strong $\mathfrak{c}$-algebrability I will compare families of cliquish functions with families of Świątkowski functions, quasi-continuous functions, Baire 1 functions and functions having the Baire property. I will also compare those families restricted to family of Darboux functions.

References

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Convergence in measure and in category

Władysław Wilczyński (University of Lodz)

The talk will present some similarities and differences concerning these two kinds of convergence. Among others convex open sets, linear functionals and bases in the space of measurable functions (or functions having the Baire property) will be considered.

Strong measure zero and selection principles

Ondřej Zindulka (Czech Technical University in Prague)

Strong measure zero sets can be characterized by the simplest selection principle S_1: A set X in a metric space is strong measure zero if and only if for each sequence $\langle \mathcal{U}_n : n \in \omega \rangle$ of uniform covers of X there is a diagonal cover of X, i.e., a sequence $U_n \in \mathcal{U}_n$ such that $\{U_n : n \in \omega\}$ covers X. It turns out that the same pattern works for other classes of sets: meager-additive sets, null-additive sets etc.

I will establish a general framework and show relations to Ramsey theory and game theory and, as an application, solve a problem of Scheepers regarding sets whose finite powers have strong measure zero.

Eggleston theorem and its generalizations

Szymon Żeberski
(Wrocław University of Science and Technology)

Our main motivation is the following result.

Theorem [Eggleston [1]] Let $A \subseteq \mathbb{R}^2$ be a Borel set of positive Lebesgue measure. Then there are two perfect sets $B, P \subseteq \mathbb{R}$ such that $B \times P \subseteq A$ and B has positive measure.

We will consider variants and generalizations of this result. In particular, we will consider various ideals on the plane of the form $\mathcal{I} \otimes \mathcal{J}$, i.e. Fubini product of $\mathcal{I}$ and $\mathcal{J}$.

Presented results are obtained together with M. Michalski (Wrocław University of Science and Technology) and R. Rałowski (Wrocław University of Science and Technology) and connected to [3] and [2].

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