p-Hierarchical Enrichment of Eigenvalue/Eigenvector Approximations

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Abstract

Many real life problems lead to challenging PDE eigenvalue problems, e.g., vibrations of structures or calculation of energy levels in quantum mechanics. A lot of research is devoted to the so-called Adaptive Finite Element Method (AFEM) which allows discretization of the governing PDE, solving the finite dimensional algebraic eigenvalue problem and iteratively improving obtained numerical approximations.

Investigating the behavior of the AFEM eigenvalue/eigenvector algorithms from the point of view of numerical linear algebra (NLA), is the scope of intensive research in the last few years. Several theoretical as well as algorithmic results clearly indicate a real necessity of engaging various NLA techniques into numerical PDE solvers, not only to obtain meaningful and relevant solutions of the real-world problems, but also to encourage the transition from hardware to algorithm oriented computational techniques. In this work, we show that an application of just one implicit inverse iteration step on the computed FEM-Ritz vector not only yields a super-converging Ritz value, but also significantly reduces the cost of underlying finite element computations. We will illustrate the ability of the presented method to solve both selfadjoint and nonselfadjoint PDE eigenvalue problems up to the accuracy guaranteed by the higher order finite elements while keeping the computational cost of the lower finite elements approximation, i.e., obtaining approximations of the $\mathbb{P}_2$ finite elements accuracy within the cost of $\mathbb{P}_1$ finite elements computations.