Normalized ground states of the nonlinear Schrödinger equation with at least mass critical growth

Bartosz Bieganowski
Institute of Mathematics, Polish Academy of Sciences (Poland)
Nicolaus Copernicus University (Poland)
bbieganowski@impan.pl

Abstract

We propose a simple minimization method to show the existence of least energy solutions to the normalized problem

$$\begin{cases} -\Delta u + \lambda u = g(u) \quad \text{in } \mathbb{R}^N, \ N \geq 3, \\ u \in H^1(\mathbb{R}^N), \\ \int_{\mathbb{R}^N} |u|^2 \, dx = \rho > 0, \end{cases}$$

where ρ is prescribed and $(\lambda, u) \in \mathbb{R} \times H^1(\mathbb{R}^N)$ is to be determined. The new approach based on the direct minimization of the energy functional on the linear combination of Nehari and Pohozaev constraints intersected with the closed ball in $L^2(\mathbb{R}^N)$ of radius ρ is demonstrated, which allows to provide general growth assumptions imposed on g. We cover the most known physical examples and nonlinearities with growth considered in the literature so far as well as we admit the mass critical growth at 0.