Highly oscillatory quadrature exploiting banded representations of differential operators

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Abstract

In this talk, we will study the efficient approximation of highly oscillatory integrals using so-called Filon and Levin methods. These are amongst the most successful methods for approximating such integrals, a task that becomes prohibitively expensive for classical quadrature.

However, Filon and Levin methods are very much topics of active research and have inherent limitations that are yet to be overcome. Perhaps a somewhat surprising connection can be made between the constructions of these methods and differential operators with banded matrix representations with respect to relevant interpolation bases.

We exploit this connection to address the important 'moment-problem' in Filon methods and hence to treat integrals arising in wave scattering that were previously not attainable in the Filon framework. Moreover, we exploit this connection to improve the efficiency of computing the Levin method, reducing the cost of approximation from $\mathcal{O}(\nu^3)$ to just $\mathcal{O}(\nu \log \nu)$ in the number of quadrature points ν. This is joint work with Arieh Iserles.