Effective high order integrators for linear Klein-Gordon equations in low to highly oscillatory regimes

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Abstract

We consider the linear Klein-Gordon equation

$$\partial_t^2 \psi(\mathbf{x},t) \quad = \quad \Delta\psi(\mathbf{x},t) + f(\mathbf{x},t)\psi(\mathbf{x},t) \tag{1}$$

with initial condition $\psi(x,0) = \psi_0(x)$, $\psi'(x,0) = \psi_0'(x)$ equipped with periodic boundary conditions (that is $x \in \mathbb{T}^d$). Here, $f(\mathbf{x},t)$ is a given, periodic in space, under the form

$$f(\mathbf{x},t) \quad = \quad \alpha(\mathbf{x},t) + \sum_n a_n(\mathbf{x},t)\mathrm{e}^{\mathrm{i}\omega n t}, \tag{2}$$

where $\omega \gg 1$ are the frequencies of the osculations. Function $\alpha(\mathbf{x},t)$ is non-oscillatory and $\sum_n a_n(\mathbf{x},t)\mathrm{e}^{\mathrm{i}\omega n t}$ is purely oscillatory function.

I will present three computational approaches to the problem:

1. Modulated Fourier based approach performing asymptotic behavior in frequencies ω;
2. Splitting methodologies resulting in second order convergence $\mathcal{O}((\Delta t)^2)$ holding uniformly in ω and fourth order convergence $\mathcal{O}((\Delta t)^4)$ under the scaling $\Delta t \gtrsim 1/\sqrt[3]{|\omega|}$;
3. Duhamel integrators of third order $\mathcal{O}((\Delta t)^3)$ uniformly in ω.

I will briefly discuss the derivation of the methods, error analysis and present plenty of numerical examples comparing derived methods against each other and against other methods known from the literature.

Results obtain in joined work with K. Kropielnicka (Institute of Mathematics, Polish Academy of Sciences), K. Schratz (Laboratoire Jacques-Louis Lions, Sorbonne Université, France), R. Perczyński, (University of Gdańsk, Faculty of Mathematics, Physics and Informatics, Poland) and M. Condon, (School of Electronic Engineering, Glasnevin, Ireland).