Parabolic equations on low-dimensional structures

Łukasz Chomienia
University of Warsaw, Poland

lc335459@students.mimuw.edu.pl

Abstract

In many applications, a need to consider certain partial differential equations on non-regular, low-dimensional structures embedded in the Euclidean space arises naturally. For example, in engineering, it is sometimes necessary to analyze a transfer of heat in conductors of umbrella-like shape or in objects of similar type, which are far from being a manifold in a classical sense. Different attempts to variational problems on low-dimensional subsets of the Euclidean space are already known. A major disadvantage of such methods is that their applicability is limited by the regularity of the structures – they cannot be applied to many interesting classes of domains. It is important to establish a consistent theory that will cover also less regular cases. Recently, a paper has been published, in which the authors establish the weak form of the stationary heat equation on a very wide class of structures, which are called multijunction measures.

The goal of my presentation is to showcase my recent results on the theory of parabolic equations on lower-dimensional subsets of $\mathbb{R}^n$. First, I am going to briefly describe how to define a second-order operator in the related setting and I am going to introduce a proper generalization of Sobolev spaces. The main part of the talk is going to be focused on the existential results, including the approach via the semigroup theory. The theorems which I am going to discuss are quite general, but it turns out that on structures of different regularity we need to realize certain proofs in an alternative way. I am going to highlight differences between structures on which the Poincaré inequality is valid and less regular ones. Besides that, I also plan to present the results on higher regularity of solutions to elliptic problems; that is, I am going to explain a correspondence between weak Sobolev solutions and those which belong to higher-order spaces.