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The modelling of a capillary rise dynamics using a nonlinear differential equation

Capillary rise is an extraordinary physical phenomenon that is ubiquitous in nature and the effects of which we meet in our everyday life. Mainly in this way, water and various nutrients are replenished in plants. Capillarity play also an important role in industry. The governing equation describing the change in the height of the liquid during the capillary flow in thin tube is a nonlinear ordinary differential equation of the second order. Due the fact that we only consider the flow in one direction (vertical tube) thus the dynamics of water height inside the tube can be successfully expressed in terms of ordinary differential equation. In my talk I am going to discuss the classical mathematical model that describes considered physical phenomena, and I will also present some of its improvements.

Furthermore, during my talk, I would like to present the most important results concerning the governing equations that models the phenomenon of capillary rise in a vertical thin tube [1, 2, 3]. In particular, I would like to present the theorem that asserts the existence and uniqueness of the solution of the discussed nonlinear differential equations. Next, I am going to present the results concerning the problem of changing the character of the capillary rise when the value of the dimensionless parameter present in the equation cross certain critical value. To complete the analysis of the dependence of the exact solution on the dimensionless parameter, the singular perturbation analysis for the limiting value of the mentioned parameter will be given.

The description of the nonlinear oscillation that occurs for some values of the nondimensional parameter may be the nontrivial example of application of the standard theories to a nonlinear problems. A proper analysis of the solution of a nonlinear equation requires the use of non-standard methods. Finally, the results presented during the talk may be helpful to better understand the features of the capillary flow and also may be useful in the study of the differential equations with the similar nonlinear component.

References

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