

A biased review of space-fractional diffusion problems

Piotr Rybka
the University of Warsaw (Poland)
rybka@mimuw.edu.pl

Abstract

We study an anomalous diffusion operator with space fractional Caputo derivative, D_x^α . The eq. takes the form,

$$\partial_t u - \partial_x D_x^\alpha u = 0$$

augmented with initial and boundary conditions.

The problem arises from modeling of flows in heterogeneous porous media. In principle it is a free boundary problem. We present existence and uniqueness results in a cylindrical domain from two different point of views. The first one is in the framework of viscosity solutions. The second one is uses the theory of analytical semigroups and eventually yields solutions to the free boundary problem.

We show continuous dependence of solutions on the fractional order of differentiation α : when $\alpha \rightarrow 0$ we obtain a transport eq.; for α tending to 1 we get the heat equation. In this context we study the question of the speed of the signal propagation.

This is a report on projects with A.Kubica, T.Namba, K.Ryszewska, V.Voller.