Shadowing of nontransversal heteroclinic chains in cubic defocusing NLS on the plane

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Abstract

In the present paper we deal with the problem of shadowing a nontransversal chain of heteroclinic connections between invariant sets (fixed points, periodic orbits, etc). The motivation for us is the work [CK+] (see also [GK]) on the transfer of energy to high frequencies in the nonlinear Schrodinger equation (just NLS from now on). From the dynamical systems viewpoint there is one remarkable feature of the construction in [CK], namely that the authors were able to shadow a non-transversal highly degenerated chain of heteroclinic connections between some periodic orbits. The length of the chain is arbitrary, but finite. Neither in [CK] nor in [GK] we were able to find a clear geometric picture showing how this is achieved, so it could be easily applicable to other systems. In this work we present a mechanism, which we believe gives a geometric explanation of what is happening. Moreover, we strive to establish an abstract framework, which will make it easier to apply this technique to other systems, both PDEs and ODEs, in questions related to the existence of diffusing orbits. The term *diffusing orbit* relates to the Arnold's diffusion for the perturbation of integrable Hamiltonian systems. We will call diffusing orbit an orbit shadowing a chain of heteroclinic connections, and occasionally the existence of such an orbit will be referred to as the diffusion.

In our picture we think of evolving a disk of dimension k along a heteroclinic transition chain and when a given transition is not transversal, then we 'drop' one or more dimensions of our disk, i.e., we select a subdisk of lower dimension "parallel to expanding directions in future transitions". After at most k transitions, our disk is a single point and we cannot continue further. We will refer to this phenomenon as the *dropping dimensions* mechanism. Since this is a new mechanism, we have found it convenient to include several figures to illustrate the main differences between transversal and non-transversal heteroclinic chains. While thinking about disks has some geometric appeal, we consider instead in our construction a thickened disk called h-set and our approach is purely topological (just as the one presented in [CK+]).

References:

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