Asymptotic expansions for PDEs with highly oscillatory input term

Rafał Perczyński
University of Gdansk (Poland)

rafal.perczynski@phdstud.ug.edu.pl

Abstract

Let us consider a well posed, linear partial differential equation with highly oscillatory input term

$$\partial_t = \mathcal{L}u + f(x,t)u(x,t), \quad t \in [0,T], \quad x \in \Omega, \tag{1}$$
$$f(x,t) = \alpha(x)e^{i\omega t}, \quad \omega \gg 1.$$

Numerical approach based on presentation of the solution as Modulated Fourier transform

$$u(x,t) \sim \sum_{r=1}^{\infty} \frac{1}{\omega^r} \sum_{n=0}^{\infty} p_{r,n}(x,t)e^{in\omega t},$$

where coefficients $p_{n,r}(x,t)$ (independent of ω) are computed numerically is a well-known and investigated tool in asymptotic numerical approach for this kind of problems. Although its efficiency has been recognised, the error analysis has not been investigated rigorously.

In this presentation I will provide analytical form of such an expansion (that is I derive analytically formulas for $p_{n,r}(x,t)$) and the rigorous form of the error term. It is the first step towards asymptotic expansions for PDEs with more general form of highly oscillatory input term

$$f(x,t) = \sum_{k=-N}^{N} \alpha_k(x,t)e^{ik\omega t}, \quad \omega \gg 1.$$

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