Optimal Hardy inequalities for the fractional Laplacian on L^p

Tomasz Jakubowski
Wrocław University of Science and Technology (Poland)

tomasz.jakubowski@pwr.edu.pl

Abstract

Let $d \geq 1$ and $0 < \alpha < d \wedge 2$. For $p \in (1, \infty)$ and $u : \mathbb{R}^d \to \mathbb{R}$ we define the p-form,

$$\mathcal{E}_p[u] := \frac{1}{2} \int_{\mathbb{R}^d} \int_{\mathbb{R}^d} (u(x) - u(y))(u(x)^{\langle p-1 \rangle} - u(y)^{\langle p-1 \rangle}) \nu(x - y)\, dy\, dx,$$

where

$$\nu(z) = \frac{2^\alpha \Gamma\big((d+\alpha)/2\big) \pi^{-d/2}}{|\Gamma(-\alpha/2)|} |z|^{-d-\alpha}, \quad z \in \mathbb{R}^d,$$

and $a^{\langle k \rangle} := |a|^k \operatorname{sgn} a$. During the talk I will discuss the following inequality

$$\mathcal{E}_p[u] \geq C \int_{\mathbb{R}^d} \frac{|u(x)|^p}{|x|^\alpha}\, dx, \quad u \in L^p(\mathbb{R}^d).$$

The explicit formula for the best constant C will be given. The talk will be based on the recent paper https://arxiv.org/abs/2103.06550