On certain estimates for a divergence form second order elliptic operator with unbounded coefficients

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Abstract

Suppose that $\mathcal{L}$ is a divergence form differential operator of the form $\mathcal{L}f(x) := e^{U(x)}/2\nabla_x \cdot \left[e^{-U(x)}(I+H(x))\nabla_x f(x)\right]$, where $U(x)$ is scalar valued and $H(x)$ is an anti-symmetric matrix valued function. We assume that they are C^2 regular but need not be bounded. We show that if $Z = \int_{\mathbb{R}^d} e^{-U(x)}dx < +\infty$ and there exists $\gamma_0 > 0$ such that $\int_{\mathbb{R}^d} e^{\gamma_0 \mathfrak{u}(x)\vee 0}\mu(dx) < +\infty$, where $\mathfrak{u}(x)$ is the supremum of the numerical range of matrix $-\nabla_x^2 U(x) + \nabla_x(\nabla_x \cdot H(x))$, then for any $1 \le p < q < +\infty$ we have $\|\nabla_x f\|_{L^p(\mu)} \le C\left(\|\mathcal{L}f\|_{L^q(\mu)} + \|f\|_{L^q(\mu)}\right)$ for $f \in C_0^\infty(\mathbb{R}^d)$. Here $d\mu = Z^{-1}e^{-U}dx$ and constant C depends only on p, q, the dimension d and γ_0. In addition, we give estimates on the spatial gradient of a semigroup $(P_t)_{t\ge 0}$ that corresponds to $\mathcal{L}$. Namely, there exist $C, t_* > 0$, depending only on p, q, the dimension d and γ_0, such that $\|\nabla_x P_t f\|_{L^p(\mu)} \le C(t\wedge t_*)^{-1/2}\left\|P_{(t-t_*)_+}f\right\|_{L^q(\mu)}$, $t > 0$.