Geometric aspects of the 1-Laplacian

Wojciech Górny
University of Vienna (Austria)
University of Warsaw (Poland)

wojciech.gorny@univie.ac.at

Abstract

The Dirichlet problem for the 1-Laplacian operator is the degenerate elliptic equation

$$-\mathrm{div}\left(\frac{Du}{|Du|}\right) = 0, \qquad u|_{\partial\Omega} = g \in L^1(\partial\Omega). \tag{1}$$

It is typically formulated as the *least gradient problem*

$$\min\left\{ \int_\Omega |Du| : \, u \in BV(\Omega), \, u|_{\partial\Omega} = g \in L^1(\partial\Omega) \right\}. \tag{2}$$

Note that equation (1) implies that the mean curvature of the level sets of a solution u vanishes. Therefore, the 1-Laplace equation is linked to the study of minimal surfaces, but also (among others) to conductivity imaging, shape optimisation, and two-dimensional optimal transport. In this talk, we focus on the relations to minimal surfaces and optimal transport in 2D to highlight the geometry behind the 1-Laplace operator. We also present the applications of methods from geometric measure theory and optimal transport to existence, regularity, and stability of functions of least gradient (i.e., solutions to (2)).