# 19th Workshop: Noncommutative Probability, Noncommutative Harmonic Analysis and Related Topics with Applications, 31.07-6.08.2022, Będlewo 


#### Abstract

Adam Paszkiewicz (common works with Stanisław Goldstein) Faculty of Mathematics and Computer Science Uniwersity of Lodz

Linear combinations of projections and perturbations of operators in von Neumann factors Abstract: We present recent results of the following type: For any hermitian operator $a \in \mathcal{M}, a \in \operatorname{lin}\left(p_{1}, \ldots, p_{n}\right)$ for some projections $p_{1}, \ldots, p_{n} \in \mathcal{M}$; for some hermitian operator $a \in \mathcal{M}$, $a \notin \operatorname{lin}\left(p_{1}, \ldots, p_{n-1}\right)$ for any projections $p_{1}, \ldots, p_{n-1} \in \mathcal{M}$. If proves that $n=4$ for $\mathcal{M}$ being a factor of type $I_{n}, n>76 ; I_{\infty} ; I I_{1}$ or $I I_{\infty}$ but $n=3$ for $\mathcal{M}$ of type $I I I$.

Similar methods gives results of type: If hermitian operators $H_{1}, H_{-1}$ satisfies $\left\|H_{+1}\right\|>1$ and some conditions in spectral language, then for any operator $0 \leqslant H \leqslant 2 \cdot \mathbf{1}$ we have $\left(H_{1}+H_{-1}^{\prime}\right)^{+}=H^{\prime}$ for some $H_{-1}^{\prime} \sim H_{-1}, H \sim H^{\prime}$. The equivalence relation $H \sim H^{\prime}$ means that $H^{\prime}=u H u^{*}$ for some partial isometry $u$ in $\mathcal{M}$.

Some new look at old methods in perturbations theory of operator in Hilbert space will also be presented.


