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ABSTRACT

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The model deformation phenomenon in free probability

Abstract: Two of the most basic objects in free probability are the circular and semicircular elements, which correspond to the large-N limits of non-Hermitian and Hermitian random matrices, respectively. The Brown measures of these elements—i.e., the large-N limits of the eigenvalue distributions of the random matrices—are uniform on the unit disk (circular law) and semicircular on [-2, 2] (semicircular law).

There is a simple but curious observation connecting these two measures: the distribution of twice the real part of the points for the circular law is the semicircular law. While this result is immediate from the formulas, it is unclear *why* there should be such a relationship.

In my talk, I will show that this connection is part of a broader phenomenon. For example, if c is circular, s is semicircular, and x_0 is a Hermitian element freely independent of c and s, then the Brown measure of $c + x_0$ is related to the law of $s + x_0$ by push-forward under a map depending only on the real part of the variable. I will describe several recent results of this sort. The talk will be self-contained and will have lots of pictures.