1-bounded entropy and embeddings into a matrix ultraproduct

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Abstract: Our work is motivated by Jung's theorem that if $\mathcal{M}$ is a tracial W*-algebra, then $\mathcal{M}$ is amenable if and only if any two embeddings of $\mathcal{M}$ into the ultrapower $\mathcal{R}^{\mathcal{U}}$ are unitarily conjugate. We show that if any two embeddings of $\mathcal{M}$ into a matrix ultraproduct $\mathcal{Q} = \prod_{n \to \mathcal{U}} M_n(\mathbb{C})$ are conjugate by an automorphism of $\mathcal{Q}$, then $\mathcal{M}$ must be strongly 1-bounded (a free entropic condition defined by Jung that means that $\mathcal{M}$ has very few matrix approximations). The proof uses a combination of Ben Hayes 1-bounded entropy and the model theory of tracial von Neumann algebras introduced by Farah, Hart, and Sherman.