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ABSTRACT

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Positive definite Q-matrices and primary non-QE graphs

Abstract: Let $G = (V, E)$ be a graph and $d(x, y)$ the graph distance. The Q-matrix of a graph is defined by $Q = Q_q = [q^{d(x,y)}]$ where $-1 \leq q \leq 1$. In spectral analysis of graphs the Q-matrix gives rise to a q-deformed vacuum functional on the adjacency algebra of G, defined by $\langle a \rangle_q = \langle Q_q e_o, a e_o \rangle$, where $o \in V$ is a fixed vertex. Bożejko [Heidelberg Lectures (1987) + private communications] proved that Q is positive definite for $0 \leq q \leq 1$ if and only if G admits a quadratic embedding (QE) in Euclidean space, and gave two basic examples of non-QE graphs, see also [Hora–Obata book (2007)]. In fact, it follows from Schoenberg's theorem (1935–37) that G is of QE class if and only if the distance matrix $D = [d(x, y)]$ is conditionally negative definite. These observations motivated us [Obata–Zakiyyah (2018)] to initiate a quantitative approach by means of the quadratic embedding constant (QEC) of a graph defined by

$$\mathrm{QEC}(G) = \max\{\langle f, Df \rangle \,;\, f \in C(V),\ \langle f, f \rangle = 1,\ \langle \mathbf{1}, f \rangle = 0\}.$$

In this talk, we discuss the recent attempt of classifying graphs in terms of the QE constants and of exploring primary non-QE graphs, that is, graphs which do not admit quadratic embeddings and do not possess such graphs as isometrically embedded subgraphs.