Motzkin cumulants

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Free cumulants are analogs of classical cumulants in the theory of free probability, in which classical independence is replaced by the concept of freeness. They are the coefficients of the R-transform of Voiculescu which plays the role of a noncommutative analog of the logarithm of the Fourier transform. From the combinatorial point of view they were defined in terms of moments by Speicher with the use of the family of lattices of noncrossing partitions. Using these lattices as well as the lattices of Motzkin paths, we introduce a hierarchy of lattices of noncrossing partitions adapted to Motzkin paths. It gives a decomposition of the combinatorics of noncrossing partitions into smaller fragments, in which depths of blocks are related to Motzkin subpaths of given Motzkin paths. Taking advantage of this hierarchy, we define path-dependent operator-valued cumulants called Motzkin cumulants and prove the corresponding Möbius inversion formula which plays the role of a decomposition of the relation between free cumulants and boolean cumulants derived by Lehner (univariate case) and Belinschi and Nica (multivariate case). More importantly, we show that it leads to a decomposition of free cumulants in terms of scalar counterparts of Motzkin cumulants.