Near-optimal randomised error for lattice rules with random number of points and deterministic generating vectors

Dirk Nuyens (KU Leuven) Laurence Wilkes (KU Leuven)
Frances Kuo (UNSW Sydney)

We focus on a new randomised algorithm for lattice-based integration. In [1], it was shown that there exists a randomised algorithm producing a lattice rule which gives the optimal rate of convergence for the worst-case expected error (randomised error) and in [2] it was shown that this rate can be achieved with an online randomised component-by-component algorithm. Fixing a budget n of maximum number of function evaluations, these two algorithms involve choosing both the number of sample points uniformly at random from the primes in $(n/2, n]$ and the generating vector at random from the "set of good generating vectors" for the specific choice of sample points. We show instead that with a single carefully selected generating vector, only the number of sample points need be chosen at random in order to achieve the optimal rate of convergence for the worst-case expected error. We show that this generating vector can be constructed in advance for a given budget n of sample points.

References

[1] P. Kritzer, F. Y. Kuo, D. Nuyens, M. Ullrich "Lattice rules with random n achieve nearly the optimal $O(n^{-\alpha-1/2})$ error independently of the dimension." *Journal of Approximation Theory* 240, 96–113, 2019.

[2] J. Dick, T. Goda, K. Suzuki "Component-by-component construction of randomized rank-1 lattice rules achieving almost the optimal randomized error rate." *Mathematics of Computation* 91, 2771–2801, 2022.