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## **Frames, designs, and discrete measures minimizing energy integrals**

While in many natural examples, energy integrals on the sphere are minimized by the uniform measure, certain interesting (especially attractive-repulsive energies) admit lower dimensional or discrete minimizers. Many interesting configurations, such as tight frames, tight spherical designs, mutually unbiased bases, optimal spherical codes etc. arise as minimizers of such energies. One of the very interesting energies with this type of behavior is the so-called  $p$ -frame energy with the potential  $F(x, y) = |\langle x, y \rangle|^p$ ,  $p > 0$ . It demonstrates very different behavior depending on whether or not  $p$  is an even integer (when  $p = 2$  this is the famous frame energy of Benedetto and Fickus). Partial results and numerical evidence suggest that minimizing measures are always discrete whenever  $p \notin 2\mathbb{N}$ . We shall survey some results and conjectures of this flavor.

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