High dimensional approximation and the curse of dimensionality

Ian H Sloan

University of NSW (UNSW Sydney)

High dimensional approximation problems commonly arise from parametric PDE problems in which an input random field depends on very many univariate random variables. Typically (for example, in the method of “generalized polynomial chaos”, or GPC) the dependence on these variables is modelled by multivariate polynomials, leading to exponentially increasing difficulty and cost (often expressed as the “curse of dimensionality”) as the dimension increases (which is why sparsity of coefficients is necessarily a major theme).This lecture makes the case, when the domain of the random variables is bounded, for using instead periodic random variables, as proposed in 2020 by Kaarnioja, Kuo and Sloan for integration, and for L_2 approximation in recent joint work with Kuo, Kaarnioja, Kazashi and Nobile. In that work the approximation is a linear combination of kernels, with the kernels located at lattice points, as advocated long ago by Hickernell and colleagues. The advantage is that the cost can grow merely linearly or quadratically with dimension, giving no cause to appeal to sparsity of coefficients. A cost comparison will be made with a GPC method based on multivariate Chebyshev polynomials of the first kind.