Infinite-Variate L^2-Approximation Based on General Linear Information using Multilevel Strategies

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We consider L^2-approximation on reproducing kernel Hilbert spaces of functions depending on infinitely many variables. This problem has been studied in [1, 2, 3], where the upper error bounds have been achieved using multivariate decomposition methods (fka changing dimension algorithms). We analyze the problem for a different cost model favorable for multilevel algorithms. The focus of the talk will be on unrestricted linear information, where we admit the evaluations of arbitrary linear functionals.
ANOVA is an acronym for "Analysis of Variance", and by ANOVA spaces we refer to function spaces whose norms are induced by an underlying ANOVA-function-decomposition. In ANOVA spaces, we adapt the analysis from [1] to prove that there is an optimal algorithm to solve the approximation problem using arbitrary information. On the other hand, for Non-ANOVA spaces, we show upper error bounds: first, by modifying a transference trick from [3] and second, by constructing a multilevel algorithm. Interestingly, the analysis reveals a performance gap between ANOVA and Non-ANOVA spaces.

[1] G. W. Wasilkowski and H. Woźniakowski. "Liberating the dimension for function approximation". In: *J. Complexity* 27 (2011), pp. 86–110.

[2] G. W. Wasilkowski and H. Woźniakowski. "Liberating the dimension for function approximation: Standard information". In: *J. Complexity* 27 (2011), pp. 417–440.

[3] G. W. Wasilkowski. "Liberating the dimension for L_2-approximation". In: *J. Complexity* 28 (2012), pp. 304–319.