Transforming approximation algorithms from the d-torus to other domains

Laura Lippert
Chemnitz University of Technology
laura.lippert@math.tu-chemnitz.de

There are several approximation operators for high-dimensional periodic functions available. We propose methods to transform functions on $\mathbb{R}^d$ or the cube $[0,1]^d$ to functions on $\mathbb{T}^d$, apply an approximation based on hyperbolic wavelet regression and transform the resulting function back. We study the problem of scattered-data approximation, where we have given sample points and the corresponding function evaluations. We transform the sample points to points on $\mathbb{T}^d$, evaluate the basis functions at these points, create a matrix and solve the matrix equation with an LSQR-algorithm to get an approximation. In our case this matrix is sparse, since we deal with compactly supported wavelets. For non-periodic functions we use an approach similar to the Fourier extension. For most approximation operators on $\mathbb{T}^d$ the error can be estimated well if we are concerned with uniformly i.i.d. samples. But with the transformation we can even deal with non-uniformly distributed data. We study two settings: the underlying density is known or unknown.

Joint work with: Daniel Potts.

References

[1] L. Lippert, D.Potts Variable Transformations in combination with Wavelets and ANOVA for high-dimensional approximation *arXiv:2207.12826*, 2022

[2] L. Lippert, D. Potts, T. Ullrich. Fast Hyperbolic Wavelet Regression meets ANOVA. *arXiv: 2108.13197*, 2021.