Sanov-type large deviations and conditional limit theorems for high-dimensional Orlicz balls

Lorenz Frühwirth and Joscha Prochno

Abstract

In this paper, we prove a Sanov-type large deviation principle for the sequence of empirical measures of vectors chosen uniformly at random from an Orlicz ball. From this level-2 large deviation result, in a combination with Gibbs conditioning, entropy maximization and an Orlicz version of the Poincaré-Maxwell-Borel lemma, we deduce a conditional limit theorem for high-dimensional Orlicz balls. Roughly speaking, the latter shows that if V_1 and V_2 are Orlicz functions, then random points in the V_1-Orlicz ball, conditioned on having a small V_2-Orlicz radius, look like an appropriately scaled V_2-Orlicz ball. In fact, we show that the limiting distribution in our Poincaré-Maxwell-Borel lemma, and thus the geometric interpretation, undergoes a phase transition depending on the magnitude of the V_2-Orlicz radius.