Constructive subsampling of finite frames and applications in function recovery

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The solution of the Kadison-Singer (KS) problem in 2015 by Marcus, Spielman, and Srivastava has led to significant progress in the search for optimal L_2 sampling schemes. The relevant consequence of KS mainly utilized in this context is Weaver frame splitting, which – iteratively applied – allows to sparsify frames of any size to optimal size subframes, i.e. frames whose size matches (in order) the underlying space dimension. In the realm of sampling reconstruction, this method can be used to generate optimal node sets filtering out the most relevant nodes from a larger sampling set. The caveat of this approach is the fact that up to now no feasible algorithms are available for the Weaver subsampling. This is the motivational point behind the research presented in this talk. Aiming for implementable algorithms, we show how a weaker result than KS, namely the BSS subsampling algorithm developed by Batson, Spielman, and Srivastava in 2009, can be adapted and extended to constructively obtain subframes similar as in the Weaver process. The method is numerically stable and polynomial in time, yet it comes with the disadvantage that the elements of the calculated subframes are weighted. However, it turns out that these weights can be controlled such that the lower frame property is essentially preserved when the weights are omitted. Applied to the subsampling of nodes, this constructive method yields nodes fulfilling optimal left Marcinkiewicz-Zygmund inequalities, which is sufficient for near-optimal sampling schemes.

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