Function Recovery and Numerical Integration on Hermite Spaces of Functions of Infinitely Many Variables

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Let us assume we want to solve a function recovery or integration problem on a tensor product reproducing kernel Hilbert space of functions depending on an infinite number of variables. The problem is computationally tractable if the variables may be arranged in such a way that their impact decays sufficiently fast. In the information-based complexity literature there are mainly two approaches to capture this phenomenon: firstly, tensor products of weighted function spaces of a fixed degree of smoothness ("weighted spaces"), where the weights moderate the importance of different (groups of) variables, and secondly, tensor products of spaces with increasing smoothness. The former type of spaces has been studied more intensively and it is well-known that (for a commonly used cost model) multivariate decomposition methods yield optimal convergence rates.
In this talk we want to explain how to use a general framework for embeddings of scales of function spaces to transfer the results for function recovery and integration established on weighted spaces to spaces of increasing smoothness. As a showcase, we study spaces of increasing smoothness that are countable tensor products of function spaces that are defined with the help of the orthonormal basis of univariate (properly normalized) Hermite polynomials h_ν, $\nu \in \mathbb{N}_0$, of $L^2(\mathbb{R}, \mu_0)$, where μ_0 denotes the standard normal distribution on $\mathbb{R}$. These "Hermite spaces" are of interest in their own right. Their norms respect the (infinite-dimensional) ANOVA decomposition, and they are intimately related to countable tensor products of spaces of Gaussian kernels (as will be discussed in more detail in a talk of Robin Rüßmann).

Part of the talk is based on the paper

[1] M. Gnewuch, M. Hefter, A. Hinrichs, K. Ritter, *Countable tensor products of Hermite spaces and spaces of Gaussian kernels*, Journal of Complexity 71 (2022), 101654.

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