Limit theorems for mixed-norm sequence spaces with applications to volume distribution

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Let $p, q \in (0, \infty]$ and $\ell_p^m(\ell_q^n)$ be the mixed-norm sequence space of real matrices $x = (x_{i,j})_{i \leq m, j \leq n}$ endowed with the (quasi-)norm $\|x\|_{p,q} := \big\| \big(\|(x_{i,j})_{j \leq n}\|_q \big)_{i \leq m} \|_p$. We shall prove a Poincaré–Maxwell–Borel lemma for suitably scaled matrices chosen uniformly at random in the $\ell_p^m(\ell_q^n)$ unit balls $\mathbb{B}_{p,q}^{m,n}$, and obtain both central and non-central limit theorems for their $\ell_p(\ell_q)$-norms. We use those limit theorems to study the asymptotic volume distribution in the intersection of two mixed/norm sequence balls. Our approach is based on a new probabilistic representation of the uniform distribution on $\mathbb{B}_{p,q}^{m,n}$.

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