Equivalence of L^2-Approximation, as well as Integration, on Gaussian Spaces and on Hermite Spaces

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We consider two types of reproducing kernel Hilbert spaces of d variables, where $d \in \mathbb{N}$ or $d = \infty$. The reproducing kernel L_σ of the first type is given as the tensor product of univariate Gaussian kernels, i.e., $L_\sigma(x, y) := \prod_{j=1}^{d} \exp(-\sigma_j^2 \cdot (x_j - y_j)^2)$, while the reproducing kernel of the second type is given as the tensor product of certain univariate Hermite kernels, i.e., $K_\beta(x, y) := \prod_{j=1}^{d} \sum_{\nu=0}^{\infty} \beta_j^\nu \cdot h_\nu(x_j) \cdot h_\nu(y_j)$, where h_v is the νth Hermite polynomial.

It turns out that if the parameters σ and β are related in a certain way, there exists an isometric isomorphism Q between $H(K_\beta)$ and $H(L_\sigma)$, such that evaluating Qf at any point only needs one point evaluation of f. We use this to study L^2-approximation and integration with respect to the d-fold product of the standard normal distribution. We obtain a one-to-one correspondence between algorithms for the L^2-approximation problem based on function evaluations on $H(L_\sigma)$ and on $H(K_\beta)$, such that corresponding algorithms have the same worst-case error and the same number of function evaluations. We get a similar result for the integration problem with a few extra steps, changing the relation between σ and β. Thus, it is possible to apply certain results for $H(L_\sigma)$ to $H(K_\beta)$ and vice versa.