On the randomized complexity of parametric integration

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Within the framework of Information-Based Complexity theory (IBC) we study parametric integration in Sobolev spaces in the randomized setting using standard information. More precisely, we determine the order of the randomized n-th minimal errors of

$$S : W_p^r(D) \to L_q(D_1), \quad (Sf)(s) = \int_{D_2} f(s,t)dt \quad (s \in D_1), \tag{1}$$

where

$$D = [0,1]^d = D_1 \times D_2, \quad D_1 = [0,1]^{d_1}, \quad D_2 = [0,1]^{d_2}, \tag{2}$$

$$1 \le p, q \le \infty, \quad d, d_1, d_2, r \in \mathbf{N}, \quad d = d_1 + d_2, \quad \frac{r}{d_1} > \left(\frac{1}{p} - \frac{1}{q}\right)_+ . \tag{3}$$

For $p = q = \infty$ such an analysis was carried out in [1] and for $1 \le p = q < \infty$ in [2].

The case $p \ne q$ requires some new techniques. Moreover, it contains a domain of parameters, namely $2 < p < q \le \infty$, where adaptive and non-adaptive randomized n-th minimal errors deviate by a power of n. Since the problem (1)–(3) is linear, this answers an old question of IBC. The result is in contrast to the deterministic setting, where it is well-known that for linear problems adaptive and non-adaptive n-th minimal errors can deviate at most by a factor of 2.

References

[1] S. Heinrich and E. Sindambiwe, Monte Carlo Complexity of Parametric Integration, *Journal of Complexity*, 15:317–341, 1999.

[2] C. Wiegand, Optimal Monte Carlo and Quantum Algorithms for Parametric Integration, Shaker Verlag, 2006.