Coproximinality of linear subspaces in generalized Minkowski spaces

Thomas Jahn
Katholische Universität Eichstätt–Ingolstadt
thomas.jahn@ku.de

Generalized Minkowski spaces (asymmetric normed spaces) extend the notion of normed spaces by *not* requiring $\|x\| = \|-x\|$ for all x. In the first part of the talk, we discuss the topological difficulties that arise from this definition. The second part of the talk is about best coapproximation. This notion was introduced by Franchetti and Furi for normed spaces by twisting the definition of best approximation. While in reflexive Banach spaces X nonempty closed convex sets K are proximal, i.e., every point $x \in X$ has a best approximation in K, an analogous property for best coapproximations does not hold true. More precisely, Franchetti and Furi show that Banach spaces of dimension ≥ 3 in which every 1-codimensional closed subspace is coproximinal are precisely Hilbert spaces. We show that this statement remains true in the context of generalized Minkowski spaces, i.e., it is independent of the symmetry of norms and of completeness.

Joint work with: Christian Richter.

References

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